Lecture 19: Nov. 7, 2013

# Quantum Bremstrahlung; Form Factors



Last time, we calculated that if an electron undergoes an impulse which changes its velocity from $\vec{v}$ to $\vec{v}\,'$, in a frame where its energy doesn't change, the expected number of photons coming off with wave number in the interval $[k, k + \Delta k]$ is

$$\frac{\alpha}{\pi}\frac{\Delta k}{k} I(\vec{v}, \vec{v}\,'),$$

with an angular distribution

$$\propto \frac{2p \cdot p'}{\hat{k} \cdot p'\, \hat{k} \cdot p} - \frac{m^2}{(\hat{k} \cdot p')^2} - \frac{m^2}{(\hat{k} \cdot p)^2}.$$

We saw that at high electron energy,

$$I(\vec{v}, \vec{v}\,') \to 2 \ln \frac{-q^2}{m^2},$$

where $q = p' - p, q^2 = -\vec{q}^{\,2}$.

More importantly, because the energy emitted in each wavenumber interval $\Delta k$ is independent of $k$, the expected number of photons diverges, both at large $k$, where we should not believe the results because our impulse approximation and the lack of loss of electron energy is unrealistic, but also as $k \to 0$, where our calculation should be correct.

Today we will calculate, to order $e^2$, the quantum-mechanical probability that a single photon of momentum $\vec{k}$ will be emitted. It is clear that each additional photon emitted takes an extra power of $e^2$, so in perturbation theory the probability is considered small even if it is multiplied by a large constant. Thus if we find a probability greater than one, it doesn't mean a mistake in our calculation itself, but just that this order in perturbation theory will be swamped by higher order terms. We will need a better way to ask our questions perturbatively than to ask what the probability that exactly one (or exactly zero) photons are emitted.

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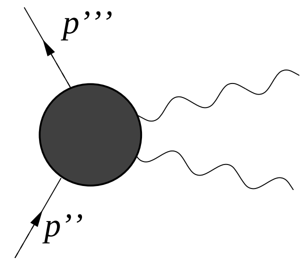


Some more comments:
In 6.20, note that $\mathcal{M}_0(p''', p'')$ is a matrix in spinor space.

Just below 6.36 we use

$$\gamma^0\gamma^i = \begin{pmatrix} -\sigma^i & 0 \\ 0 & \sigma^i \end{pmatrix}$$

At the bottom of page 187 we are using

$$\begin{aligned} \bar{u}\sigma^{i0}u &= -iu^\dagger \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix} u \sim 0 \\ \bar{u}\sigma^{ij}u &= \epsilon_{ijk}u^\dagger \begin{pmatrix} 0 & \sigma^k \\ \sigma^k & 0 \end{pmatrix} u \sim 2m\epsilon_{ijk}\xi^\dagger\sigma^k\xi. \end{aligned}$$

On page 188, note that if the classical Maxwell field

$$A^{\rm cl}(x) = \int \frac{d^4k}{(2\pi)^4} e^{-ik\cdot x}\tilde{A}(k),$$

then

$$\widetilde{\nabla \times A}(k) = -i\vec{k} \times \tilde{\vec{A}}(k).$$

Read Peskin and Schroeder pp. 182–189, and also “Schwinger Trick and Feynman Parameters”, from the supplementary material posted on the web.