Faddeev-Popov ghosts

In evaluating the functional integral over gauge fields, we found that we could add a gauge-fixing term to the Lagrangian which would make the integrals better defined and evaluatable in perturbation theory, but that in order to do so, a new factor appeared in the integrand. This new factor, the Faddeev-Popov determinant, is

$$\Delta^{-1}[\mathcal{A}] = \det \frac{\delta\, \partial^\mu (A^\lambda_\mu)^{(b)}}{\delta \lambda^{(a)}}.$$

We found that for Abelian gauge theory this factor is independent of all the fields and is therefore an ignorable constant. But this is not the case in non-Abelian theory. We now turn to evaluating this factor.

Recall that the Faddeev-Popov determinant is gauge invariant, so we can evaluate it for a field satisfying the Lorentz condition, $\partial^\mu \mathcal{A}_\mu = 0$, and therefore the gauge transformations considered will be infinitesimal, and $(A^\lambda_\mu)^{(b)} = A^{(b)}_\mu + g^{-1}(D_\mu \lambda)^{(b)}$, The Faddeev-Popov determinant is therefore the determinant of g^{-1} times the operator $\partial^\mu D_\mu$, where the covariant derivative is to use the adjoint representation suitable for λ