Physics 615 Fall, 2013

Homework #3 Due: Sept. 27 at 3:00 P.M.

1) In lecture we defined the Pauli-Lubanski vector

$$W^\mu = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma} P_\nu L_{\rho\sigma},$$

built of the generators of the Poincaré group. Show that

(a) $[W^\mu, P_\nu] = 0$,

find $[W^\mu, L_{\nu\rho}]$, and then show W^2 is a Casimir operator of the Poincaré group, that is

(b) $[W^\mu W_\mu, P_\nu] = 0$,

(c) $[W^\mu W_\mu, L_{\alpha\beta}] = 0$.

Also show, if you haven't already, that P^2 is also a Casimir operator of the group.

It may be useful to examine whether your evaluation of $[L_{\alpha\beta}, W^\mu]$ is what you would expect for a general vector, $[L_{\alpha\beta}, V^\mu]$, and also to show that the vector properties implied by the indices do transform correctly under commutator with $L_{\alpha\beta}$. That is, $V^\mu F_\mu$ should commute, and the c-numbers $g_{\mu\nu}$ and $\epsilon_{\mu\nu\rho\sigma}$, which of course do commute with all operators, **should** commute with Lorentz transformations despite having Lorentz indices.

2) Consider a single real free scalar field with the Klein-Gordon Lagrangian.

(a) Find the expression for $T^{\mu\nu}$ in terms of ϕ, π, and $\nabla\phi$. From this plugged into $\mathcal{M}^{\mu\nu\rho}$, find

(b) $\vec{J}(t)$

(c) $\vec{K}(t)$

as integrals over products of $\phi(\vec{x})$ and $\pi(\vec{x})$. [Recall $\vec{J} = (L_{23}, L_{31}, L_{12})$ and $\vec{K} = (L_{01}, L_{02}, L_{03})$.] From these results, find the values of

(d) $[P^\mu(t), \phi(\vec{x}, t)]$,

(e) $[\vec{J}(t), \phi(\vec{x}, t)]$

(f) $[\vec{K}(t), \phi(\vec{x}, t)]$

at equal times, in terms of $\phi(\vec{x})$ and its derivatives.