Physics 615 Wed. Dec. 11, 2013
Homework #12

1 [5 pts] Consider a theory in which the matter fields consist of a single complex field, $\Phi(\mathbf{x})$, equivalent to two real fields, with gauge transformations given by a single real scalar field $\lambda(\mathbf{x})$ under which $\Phi(\mathbf{x}) \to e^{i\lambda(\mathbf{x})}\Phi(\mathbf{x})$.

- a) What gauge fields do we have and what are their transformations under the gauge transformation above?
- b) Find the expression for $F^{\mu\nu}$ in terms of the gauge field.
- c) How does the field strength $F^{\mu\nu}$ change under a gauge transformation?

2 [5 pts] Now consider a theory (of "weak isospin") in which the gauge group is SU(2), with a three-dimensional Lie Algebra identical to the rotation group,

$$[L_i, L_j] = i\epsilon_{ijk} L_k,$$

where ϵ_{ijk} is the three dimensional Levi-Civita symbol, defined by $\epsilon_{123} = 1$ and by being totally antisymmetric in its three indices. Let the matter fields be described by a pair of **complex** fields transforming under a complex two dimensional representation with

$$M(L_k) = \frac{1}{2}\sigma_k.$$

Answer the same questions as above. For finite gauge transformations simplifying will not be easy and is not required, but for infinitesimal gauge transformations, give the answers in terms of the components (that is, with the L_j eliminated).

3 [5 pts] In lecture I showed that the equations of motion for a gauge theory require

$$(D_\rho F^{\rho\mu})^{(a)} = -g j^{a\,\mu},$$

where $j^{a\,\mu}$ is the current, which is now Lie-algebra valued. From this equation, show that the current is conserved using the covariant derivative:

$$(D_\mu j^\mu)^a = 0.$$

[Note: you need to be careful with what the covariant derivative means. Recall that it has a gauge field multiplied by the representation matrix for whatever it acts on. Here it is acting on Lie-algebra valued field, so the appropriate representation is the adjoint.]