Physics 613 Lecture 24 April 29, 2014

SU(2)×U(1) Gauge Theory of Electroweak Interactions

In the last lecture we described the gauge group for the electroweak interactions as SU(2)×U(1), broken by the complex Higgs doublet. We saw that the neutral gauge particles W^0 and B mix, leaving an unbroken gauge symmetry for electromagnetism and a neutral Higgs.

Today we will introduce the quarks and leptons, so we can actually describe electromagnetic and weak interactions.

Fortunately, the doublet nature of weak interactions was apparent long before the sophisticated nature of our model was known. The weak current, being charged, connected pairs of particles with charges differing by one unit, and in fact looked like a collection of doublets, whether proton-neutron or $\nu_e - e^-$ or $\nu_\mu - \mu^-$ or $u - d$ quarks. The first and last, of course, are the origins of hadronic isospin, and are only approximately the doublets we are going to want for weak isospin. But all of the fundamental matter particles we will want to introduce will be spinors, and their left-handed pieces will be doublets under the SU(2) gauge, while their right-handed pieces will be singlets. These spinors will be massless in the fundamental lagrangian, as we can't have a gauge-invariant mass term coupling a doublet and a singlet. There will be three generations of leptons and three generations of quarks. The doublets all interact with the $\vec{W}_\mu$ with the strength $g\vec{\sigma}/2$, while the singlets, of course, do not. The upper component of each doublet has a charge one unit higher than the lower component, so $\Delta Q = \Delta t_3$, where t and t_3 are the weak isospin and its third component. All multiplets interact with B_μ with a strength $ig'y/2$, where the *weak hypercharge* y varies from multiplet to multiplet, -1 for the left-handed leptons, -2 for the right-handed negatively charged leptons, 0 for the right-handed neutrinos (if they exist). The left-handed quarks have $y = 1/3$, while the right-handed quarks, in order to have the same charge as their left-handed components, have $y = 4/3$ for u, c, and t, and $y = -2/3$ for d, s, and b. Note in all cases the charge Q (in units of $e > 0$) is

$$Q = t_3 + \frac{1}{2}\,y.$$

Thus each of the spinor fields enters the lagrangian with

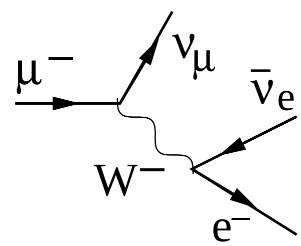
$$\mathcal{L} = i\bar{\psi}\not{D}_L\frac{1-\gamma_5}{2}\psi + i\bar{\psi}\not{D}_R\frac{1+\gamma_5}{2}\psi$$

with

$$\begin{aligned} D_{L\,\mu} &= \partial_\mu + ig\vec{\sigma}\cdot\vec{W}_\mu/2 + ig'y\mathbb{1}B_\mu/2, \\ D_{R\,\mu} &= \partial_\mu + ig'yB_\mu/2. \end{aligned}$$

though for the right-handed neutrinos, as their $t = y = 0$, there is no way for them to interact and no way for us to know, in the standard model, whether or not they exist!

Let us consider the decay of the muon, $\mu^- \to e^- + \nu_\mu + \bar{\nu}_e$, This proceeds by the second order interaction with W exchange. We note that $\vec{\sigma}\cdot\vec{W}_\nu = \frac{1}{\sqrt{2}}(\sigma_+ W_\nu + \sigma_- W_\nu^\dagger)$ where $\sigma_\pm = \frac{1}{2}\sigma_1 \pm \sigma_2$ and $W^\nu = (W_1^\nu - iW_2^\nu)/\sqrt{2}$. The reason for this notation is that $\sigma_- = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$ and W has the right weight for a propagator. Now the relevant part for the electron field is σ_- and for the muon σ_+, so the invariant amplitude is



$$i\mathcal{M} = \left(i\frac{g}{\sqrt{2}}\bar{\psi}_e\gamma^\rho(1-\gamma_5)\psi_{\nu_e}\right)\frac{i(-g_{\rho\tau}+k_\rho k_\tau/M_W^2)}{k^2-M_W^2}\left(i\frac{g}{\sqrt{2}}\bar{\psi}_{\nu_\mu}\gamma^\tau(1-\gamma_5)\psi_\mu\right).$$

But the muon only has a mass of 105 MeV so the momentum transfer $k \ll M_W = 80.38$ GeV, so we can pretty well put k to zero, and

$$i\mathcal{M} = i\frac{g^2}{2M_W^2}\bar{\psi}_e\gamma^\rho(1-\gamma_5)\psi_{\nu_e}\bar{\psi}_{\nu_\mu}\gamma_\rho(1-\gamma_5)\psi_\mu$$

which is exactly the old Fermi four-fermion interaction with coupling constant

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2} = 1/2v^2.$$

From the long-measured value $G_F = 1.166\times10^{-5}\,\mathrm{GeV}^{-2}$ we find $v \approx 246$ GeV.

The same diagram, charge conjugated, will give the scattering cross section for $\nu_\mu + e^- \to \nu_e + \mu^-$, as in Fig. 22.10 in the book, which might be used to detect muon neutrinos. But it was also discovered in bubble chambers in 1973 that there was also an elastic cross section, $\nu_\mu + e^- \to \nu_\mu + e^-$, which must proceed by a neutral current, the Z. From the effective coupling constant here, together with the electric charge $= g\sin\theta_W$, we have the three experimental values we need to determine g, g' and v, and thus

$$\sin^2\theta_W = 0.231, \qquad M_Z = M_W/\cos\theta_W = 91.2\,\text{GeV}.$$

Quark Weak Interactions

As we mentioned, all the fundamental spinors have their left-handed components as part of a weak isodoublet, and their right handed components immune to the $\vec{W}$ gauges, feeling only the B in their covariant derivatives. Thus the left handed components are

$$\left\| \begin{array}{cccc|ccc} & \begin{pmatrix}\nu_e\\ e^-\end{pmatrix} & \begin{pmatrix}\nu_\mu\\ \mu^-\end{pmatrix} & \begin{pmatrix}\nu_\tau\\ \tau^-\end{pmatrix} & \begin{pmatrix}u\\ d'\end{pmatrix} & \begin{pmatrix}c\\ s'\end{pmatrix} & \begin{pmatrix}t\\ b'\end{pmatrix} \\ y = & -1 & -1 & -1 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{array} \right\|$$

all with $t = 1/2$. The right handed quarks and leptons do not come in doublets. The leptons e^-_R, μ^-_R, τ^-_R have $y = -2$, the quarks whose mirror images are $t_3 = +1/2$ (u_R, c_R, t_R) have $y = 4/3$ and the others $y = -2/3$ so as to have total electromagnetic charges of $2/3$ and $-1/3$ respectively.

We have distinguished between hadronic isospin and weak isospin, but we haven't discussed what that difference is. When we classify the u and d as a hadronic isospin doublet, we are basing the assignment of what particle comes from $T_-\,|u\rangle$ on the overwhelming strength of the strong interaction. But what about $t_-\,|u\rangle$, the action of the weak isospin? It gives us a quark of charge $-1/3$, but might it not be a mixture of the d, s and b quarks as distinguished by strong interactions and their very different masses? We have written the weak isodoublets in terms of primed quark fields to distinguish them from the strong interaction conserved flavors. We did not bother with the upper components, because we can choose the weak upper components to be whatever the strong interactions wanted by definition of which generation is which. But then the lower components are what the W's produce from the upper ones. These three fields, d', s' and b' must be a unitary transformation of the mass eigenstates d,s and b, so we may write

$$\begin{pmatrix}d'\\ s'\\ b'\end{pmatrix} = \mathbf{V}\begin{pmatrix}d\\ s\\ b\end{pmatrix} \qquad \text{with} \quad \mathbf{V} = \begin{pmatrix}V_{ud} & V_{us} & V_{ub}\\ V_{cd} & V_{cs} & V_{cb}\\ V_{td} & V_{ts} & V_{tb}\end{pmatrix}$$

where $\mathbf{V}$ is the CKM or Cabbibo, Kobayashi, Maskawa matrix.

Note that if there were no such discrepancy between weak and strong isospin, that is, if $\mathbf{V}$ were $= 1\!\!1$, there would be conservation of (strangeness + charm), and K mesons could not decay. So part of the CKM matrix, V_{us}, was proposed by Cabbibo in 1963 before three generations or even the charmed quark were known.

For the leptons, as far as we know[1], each generation does have its own conservation law, so there is no mixing, and so only the leptonic equivalent of V_{ud}, V_{cs}, V_{tb} are nonzero, and are pure phase transformations. Also, because the phase of each field is arbitrary, we can adjust phases to make the leptonic equivalent of $\mathbf{V}$ be the identity. For the quarks, we can adjust the six phases of the weak and strong fields, but adjusting them all by the same amount leaves $\mathbf{V}$ unchanged, so there are 5 adjustable parameters of the nine real parameters that a 3×3 unitary matrix has. Thus there are four parameters which need to be measured to know $\mathbf{V}$.

One important consequence of this possible mixing is that the $\mathbf{V}$ matrix cannot be made real by the choice of field phases. If it could, it would be an orthogonal 3×3 matrix with only three adjustable parameters. That there is room in $\mathbf{V}$ for a complex parameter which cannot be made real means that CP conservation, which is equivalent to T conservation in QFT, can be violated, as time reversal involves complex conjugation. If there were only two generations of quarks, this would not be the case, and the weak interactions would not give an explanation of CP violation, such as seen in the $K^0 - \bar{K}^0$ system and also the corresponding D^0 and B^0 systems.

Fermion Masses

We have seen that the left-handed and right-handed parts of the spinors transform differently under weak isospin, so a simple mass term $m\bar{\psi}\psi$ is not invariant. This is why the theory we are using, before symmetry breaking, has no massive spinors.

But the vacuum expectation value ϕ_0, which breaks the symmetry, provides a mechanism for generating mass. Under weak isospin, $\bar{\psi}_L$ transforms

[1] Wrong! Wrong! Wrong! We now have conclusive evidence for neutrino mixing, which means one flavor of neutrino can turn into a different one, so there is no conservation of the number of leptons of each individual flavor. It is still not known if there is conservation of the total number, $N_e + N_\mu + N_\tau$.

properly[2] so that $\bar\psi_L\phi$ is a scalar under weak isospin. So

$$\mathcal{L}^{(e)}_{\rm Yuk} = -g_e\bar\psi_{e,L}\phi\psi_{e,R} + h.c.$$

obeys all required symmetries, and reduces to $-\dfrac{g_e v}{\sqrt{2}}\left(\bar e_L e_R + \bar e_R e_L\right) = -m_e\bar\psi_e\psi_e$, when we discard terms cubic in the fluctuating fields, replacing ϕ by ϕ_0, This provides the electron with a mass $m_e = g_e v/\sqrt{2}$. Thus we see that the vacuum breakdown provides mass to the fermions as well as the $W^\pm$, provided there is a Yukawa coupling in the Lagrangian of the electron with the Higgs field. But this means there is also a coupling to the fluctuating part of the Higgs field, $-\dfrac{g_e v}{\sqrt{2}}\hat h(x)\left(\bar e_L(x)e_R(x) + \bar e_R(x)e_L(x)\right) = -\dfrac{g m_e}{2M_W}\hat h(x)\bar e(x)e(x)$.

The other bottom components of the isodoublets can develop masses in the same way as the electron. But what about the upper components? The secret comes from closer examination of the transformation of the standard isospin 1/2 representation, under which

$$\psi_a \to \left(e^{i\vec\alpha\cdot\vec\sigma/2}\right)_{ab}\psi_b.$$

The conjugate representation has

$$\psi^\dagger_a \to \left(e^{-i\vec\alpha\cdot\vec\sigma^T/2}\right)_{ab}\psi^\dagger_b.$$

Thus it does not transform like the standard representation, but $\chi_a := \epsilon_{ac}\psi^\dagger_c$ does, where $\epsilon_{ab} = (i\sigma_2)_{ab}$ is the two-dimensional antisymmetric Levi-Civita tensor $\epsilon_{12} = 1 = -\epsilon_{21}$, $\epsilon_{11} = \epsilon_{22} = 0$ in isospin space. To see this,

$$\chi_a \;:=\; \epsilon_{ac}\psi^\dagger_c \to \left[(i\sigma_2)\left(e^{-i\vec\alpha\cdot\vec\sigma^T/2}\right)\psi^\dagger\right]_a = \left[\left(e^{i\vec\alpha\cdot\vec\sigma/2}\right)(i\sigma_2)\,\psi^\dagger\right]_a = \left(e^{i\vec\alpha\cdot\vec\sigma/2}\chi\right)_a.$$

So $\epsilon_{ab}\bar\psi_{u,L\,b}$ transforms like ϕ_a and $\phi^\dagger_a\epsilon_{ab}\bar\psi_{u,L\,b}$ is invariant, and can be contracted with $\psi_{u,R}$. Of course $\phi^\dagger_{0\,a}\epsilon_{ab} = (\frac{v}{\sqrt{2}}, 0)$. Thus the $u-d$ quark Yukawa interaction is $\mathcal{L} = -\lambda_d\bar\psi_{u,L}\cdot\phi d_R - \lambda_u\bar\psi_{u,L}(i\sigma_2)\phi^\dagger u_R + h.c.$

Thus we see that we can introduce any spinor masses to the quarks and the charged leptons we wish, so our theory can accommodate any quark and lepton masses, but does not predict them. In so doing, the couplings of these

[2] The convoluted language, rather than saying $\bar\psi_L$ is $t = 1/2$, is because the conjugate to a standard $t = 1/2$ representation is *equivalent* to the standard representation, it is not equal to it.

fermions to the higgs field is determined, with a strength proportional to the induced fermion mass. Thus you might think the experimentalists found the Higgs by looking for $b\bar{b}$ and their decays, but unfortunately that channel has so much background this is impossible to extract. It was actually the coupling to the gauge particles, photons and Z's, together with $\tau\bar{\tau}$, that were detected.