Physics 613 Lecture 23 April 24, 2014

Higgs Mechanism

Last time we discussed spontaneous symmetry breaking, where a theory with a multiplet of scalar fields ϕ transforming as a unitary irreducible representation of a symmetry group $\mathcal{G}$. The Lagrangian was invariant under $\mathcal{G}$, but possibly the lowest energy state (vacuum), about which we do perturbation theory, was not invariant. The vacuum expectation value ϕ_0 may be left invariant under a subgroup $\mathcal{K} \subset \mathcal{G}$, and the dimensions in the Lie Algebra $\mathfrak{G}$ of $\mathcal{G}$ which do not leave ϕ_0 invariant ($\mathfrak{G}/\mathfrak{K}$, generators of the cosets of $\mathcal{G}/\mathcal{K}$) give rise to massless scalar particles known as Goldstone Bosons.

In the previous two lectures we considered theories with multiplets, whether scalar fields ϕ or fermions ψ, which transform as unitary irreducible multiplets under a symmetry group $\mathcal{G}$ which we made local by replacing ordinary partial derivatives by covariant derivatives, depending on new gauge fields taking values in $\mathfrak{G}$. We saw that this necessitated introducing massless vector particles in the adjoint representation of the group. Had we had more time, we would have discussed the problems of ensuring that the propagator doesn't give rise to unphysical states, which for Abelian theory was assured by the Ward identity. This requires the gauge particles to be massless, and when we considered the photon propagator, we argued that radiative corrections would not give the photons a mass because there are no massless scalar particles which could give a pole in $\Pi(q^2)$ at $q^2 = 0$.

But today we will combine our two ideas, and as the Goldstone bosons **are** scalar particles with mass zero, we may find a very interesting effect called the Higgs mechanism. This is the way we now understand the electroweak interactions, which, together with the color gauge theory of QCD, the strong interactions, forms the basis of the standard model of elementary particle physics.

The Abelian Higgs Model

Our first example will be a simple U(1) theory with a complex scalar ϕ. As with electromagnetism, this means we have one gauge field A_ν and a local phase symmetry[1] $\phi \to e^{-i\alpha(x)}\phi$, $A_\nu \to A_\nu + \frac{1}{q}\partial_\nu \alpha(x)$. We take the lagrangian

[1]Eq. 19.41 is wrong.

density to be

$$\mathcal{L} = (D^\nu\phi)^\dagger D_\nu\phi - \frac{1}{4}F_{\nu\rho}F^{\nu\rho} + \mu^2\phi^\dagger\phi - \frac{\lambda}{4}(\phi^\dagger\phi)^2. \tag{1}$$

Were it not for the mass term having the wrong sign, this would be simply a theory of a charged scalar interacting with photons. There would be the two scalar degrees of freedom from ϕ, and the two transverse polarizations of the photon, for a total of four. But note that the mass term does have the wrong sign, so we will have spontaneous symmetry breaking. The classical minimum of the potential has $\phi^\dagger\phi = \frac{2\mu^2}{\lambda}$, so we choose our vacuum to be at $\phi = v/\sqrt{2}$ where v is real[2] and $v = 2\mu/\sqrt{\lambda} > 0$. We need to reexpress the two ϕ degrees of freedom. In the book, rather than doing this with cartesian coordinates, they write ϕ in terms of a radial coordinate coordinate $|\phi| = v + h(x)$ and an angular coordinate or phase angle θ/v. Thus

$$\sqrt{2}\hat{\phi}(x) = \left(v + \hat{h}(x)\right) e^{-i\hat{\theta}(x)/v}.$$

This has the effect of introducing a term *linear* in the field $\hat{\theta}$ into the electromagnetic current, which means there is a quadratic term in $A_\nu j^\nu$. There is also one from the $-2q^2 A\phi^\dagger\phi$ which comes from the usual $(D^\nu\phi)^\dagger D_\nu\phi$ term. Though this is cubic in unbroken fields, the vacuum expectation value leaves it with a linear piece $-q^2v^2A$. These mean the quadratic part of the Lagrangian that determines the A propagator has additional A dependent terms which modify the propagator.

’t Hooft Gauge

There is another way to view this effect. Let us choose a different approach, using cartesian coordinates, $\sqrt{2}\,\phi(x) = v + \chi_1(x) + i\chi_2(x)$, so $2\phi^\dagger\phi = (v+\chi_1)^2 + \chi_2^2$, and, replacing μ^2 by $v^2\lambda/4$, we have

$$\begin{aligned} V(\phi) &= -\frac{v^2\lambda}{8}\left[v^2 + 2v\chi_1 + \chi_1^2 + \chi_2^2\right] + \frac{\lambda}{16}\left[v^2 + 2v\chi_1 + \chi_1^2 + \chi_2^2\right]^2 \\ &= -\frac{v^4\lambda}{16} + (-1{+}1)\frac{v^3\lambda}{4}\chi_1 + (-4+6)\frac{v^2\lambda}{16}\chi_1^2 + (-1+1)\frac{v^2\lambda}{8}\chi_2^2 + \mathcal{O}(\phi^3) \\ &= -\frac{v^4\lambda}{16} + \frac{v^2\lambda}{8}\chi_1^2 + \mathcal{O}(\phi^3) \end{aligned}$$

[2]The funny $\sqrt{2}$ factors are because we define $\sqrt{2}\,\phi = \phi_1 - i\phi_2$, so $\phi_{j,0} = (v, 0)$.

Note the constant term $-v^4\lambda/16$ is irrelevant, the χ_1 term develops a mass $m^2 = v^2\lambda/4 = \mu^2$, and χ_2 becomes massless.

But we are not done examining the quadratic pieces of the Lagrangian, because

$$\begin{aligned}(D^\nu\phi)^\dagger D_\nu\phi &= [(\partial^\nu + iqA^\nu)\phi]^\dagger(\partial^\nu + iqA^\nu)\phi \\ &= \frac{1}{2}(\partial_\nu\chi_1)^2 + \frac{1}{2}(\partial_\nu\chi_2)^2 + qvA^\nu\partial_\nu\chi_2 + \frac{1}{2}q^2v^2A^\nu A_\nu \\ &\qquad + \text{terms cubic and higher order in the fields.}\end{aligned}$$

The last term gives a mass $m_A^2 = q^2v^2$ to the space components $\vec{A}$.

But what about the Ward identity, or alternately, what about the time component, for which the mass term seems to have the wrong sign? If we treat the mass as an interaction $im_A^2 g^{\mu\nu}$ we might worry about the first diagram, but there is another diagram in which the photon interacts with the χ_2 particle with a vertex $m_A k^\mu$, two of which, together with the massless χ_2 propagator, gives for the second diagram $(m_A k^\mu)\frac{i}{k^2}(-m_A k^\nu)$, so the two diagrams combine to give

$$im_A^2\left(g^{\mu\nu} - \frac{k^\mu k^\nu}{k^2}\right),$$

which kills the component in the k^μ direction, or, in the rest frame, the time component of the spin.

Because we know that this theory has a local gauge invariance under phase transformations, given any field configuration, we may choose our gauge transform so as to undo the phase of ϕ, setting $\theta = 0$. This does away with the degree of freedom associated with the massless χ_2. But this degree of freedom has not disappeared, because it has been consumed by the A field, which has become massive and now has three degrees of freedom as a massive spin one particle should.

We have worked this out for the simple case of one Abelian gauge field, but we should note that the gauge fields that might pick up mass and the Goldstone bosons that can be eaten by them share a common feature — they are both associated with directions in the Lie algebra. The original multiplet of scalars was in a representation of the algebra but not necessarily the adjoint, but the Goldstone bosons are directly coupled to the Lie algebra generators which are broken by the vacuum.

Broken Non-Abelian Gauge Theory

Let us consider the simplest non-Abelian example, with the group $\mathcal{G}$ =SU(2). This has a three-dimensional Lie algebra and so we begin with three gauge particles A^a_μ. Our scalar particles will need to transform under some representation of SU(2). First let's consider the isospin 1/2 representation, a complex doublet ϕ. The algebra is represented by $T_a = \sigma_a/2$, so the covariant derivative is

$$D_\mu\phi = (\partial_\mu - iqA^a_\mu T_a)\phi.$$

With the ϕ in (1) now refering to this complex doublet, we still see the minimum of the potential requires $2\phi^\dagger\phi = v$, but now we choose ϕ_0 not only to be real but to have zero upper component,

$$\phi_0 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix}.$$

Then the $|D_\mu\phi|^2$ term quadratic in A is

$$\frac{g^2}{2}(0,v)T_aT_b\begin{pmatrix} 0 \\ v \end{pmatrix} A^a_\mu A^{b\,\mu} = \frac{g^2v^2}{8}A^a_\mu A^{a\,\mu},$$

where the symmetry of $A^a_\mu A^{b\,\mu}$ under $a \leftrightarrow b$ enabled us to replace T_aT_b by $\frac{1}{2}\{T_a, T_b\} = \frac{1}{8}\{\sigma_a, \sigma_b\} = \delta_{ab}/4$. Thus each of the three gauge particles develops a mass $m_A = gv/2$, and there are no massless vectors left. We will have one remaining massive real scalar left, coming from the oscillations around $v/\sqrt{2}$ of the real part of the lower component of the doublet. The other degrees of freedom have been eaten, fixed to be zero by choice of gauge (called unitary gauge).

Now we might have chosen ϕ to transform differently. For example, we might have chosen an isospin 1 real field, with three real components. This transforms under the adjoint representation of SU(2), so now

$$(D_\mu\phi)_a = \partial_\mu\phi_a + g\epsilon_{abc}A^b_\mu\phi_c,$$

and the A^2 term in the lagrangian comes from $\frac{1}{2}(D_\mu\phi)^2 = \frac{g^2}{2}(\epsilon_{abc}A^b_\mu\phi_{0\,c})^2$. The theory has spherical symmetry, so if the $V(\phi)$ term takes its minimum value for $|\phi| = v \neq 0$, it can be anywhere on a sphere of radius v, and we can choose that to be along the third axis, so $\phi_0 = (0, 0, v)$, and $(\epsilon_{abc}A^b_\mu\phi_{0\,c})^2 = v^2\epsilon_{ab3}A^b_\mu\epsilon_{ac3}A^{c\,\mu} = v^2[(A^1_\mu)^2 + (A^2_\mu)^2]$. Thus two of the gauge particles develop masses, $m_1 = m_2 = gv$, but the third remains massless. And ϕ_3 becomes the sole surviving massive real scalar.

In both of these examples, any point that could be the minimum of $V(\phi)$ was equivalent to any other under the symmetry, but that is not always the case. One example[3] is an SU(3) gauge group with an octet scalar. If ϕ_0 is in the λ_8 direction, the SU(3) is broken into SU(2)×U(1), so those four gauge particles remain massless, while the other four develop equal positive masses. But if ϕ_0 is in the λ_3 direction, only A^3 and A^8 remain massless, the symmetry is broken to U(1)×U(1), and four of the other vector particles develop a mass M and the other two a mass $2M$.

A Side Comment on g

We saw in homework 10 that if the Killing form is positive definite, it presented a natural way to normalize the basis vectors of the Lie algebra. This gives a natural metric in group space, and such groups are compact sets, so they have a natural size. But the symmetries act linearly on the scalar or spinor fields, so there is no natural strength by which a gauge field should act on a matter field, so we have a parameter, a kind of charge, g, which we have always seen in our covariant derivatives of matter fields. The strengths by which the different gauge fields act on the matter fields is, however, determined by the matter-field representation, if it is irreducible.

If, however, the gauge group is a direct product of two groups, the covariant derivative will be a sum over gauge fields from the two different groups, and the strength with which each couples will not be constrained. So there will be separate coupling constants for the two components.

SU(2)×U(1) Gauge Theory with Isodoublet Higgs

Now let us consider the group which will give us the Glashow-Salam-Weinberg model of the electroweak interactions, which is a major component of the standard model. The group is SU(2)×U(1). The gauge particles are three $\vec{W}_\mu$'s for SU(2) and one B_μ for U(1). The field strength for the W's will be called F

$$\vec{F}_{\mu\nu} = \partial_\mu \vec{W}_\nu - \partial_\nu \vec{W}_\mu - g\vec{W}_\mu \times \vec{W}_\nu,$$

[3] Peskin and Schroeder, pp. 696-697.

and that for B will be called G,

$$\vec{B}_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu.$$

All of the particle fields we know about, quarks and leptons, will be included, but we concentrate for now on a doublet of complex scalars ϕ which will spontaneously break the symmetry. Acting on ϕ, the covariant derivative is

$$D_\mu = \partial_\mu + i\frac{g}{2}\vec{\sigma}\cdot\vec{W}_\mu + i\frac{g'}{2}B_\mu.$$

We are looking to have vector fields with charge, so the doublet needs to have different charges for its two components, and we want the one that develops a vacuum expectation value to be neutral, so we write $\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$ and $\phi_0 = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$. The general field configuration for ϕ can now be written as

$$\sqrt{2}\,\hat{\phi} = e^{-i\vec{\theta}\cdot\vec{\sigma}/2v}\begin{pmatrix} 0 \\ v+\hat{h}(x) \end{pmatrix},$$

but we will immediately go to the unitary gauge, which undoes the exponential factor, leaving from $\hat{\phi}$ only the real scalar Higgs field $h(x)$ with a mass $\sqrt{2}\,\mu = v\sqrt{\lambda/2}$. But the $|D_\mu\phi|^2$ term now gives us a term

$$\frac{1}{8}(0,v)[g\vec{\sigma}\cdot\vec{W}_\mu + g'B_\mu]^2\begin{pmatrix} 0 \\ v \end{pmatrix} = \frac{v^2}{8}\left[g^2(\hat{W}_1^2+\hat{W}_2^2) + (gW_3 - g'B)^2\right].$$

We see that $m_{W_1} = m_{W_2} = gv/2$, but the mass matrix is not diagonalized by our choice of basis vectors for the other two gauge fields, and we need to choose a new basis by rotating in the $W_3{-}B$ plane,

$$\begin{aligned}\hat{Z}^\mu &= \cos\theta_W \hat{W}_3^\mu - \sin\theta_W \hat{B}^\mu \\ \hat{A}^\mu &= \sin\theta_W \hat{W}_3^\mu + \cos\theta_W \hat{B}^\mu\end{aligned}$$

where

$$\cos\theta_W = \frac{g}{\sqrt{g^2+g'^2}}, \qquad \sin\theta_W = \frac{g'}{\sqrt{g^2+g'^2}}.$$

Then the gauge field mass terms are $\frac{1}{2}M_W\,(W_1^\mu W_{1\,\mu} + W_2^\mu W_{2\,\mu}) + \frac{1}{2}M_Z Z^\mu Z_\mu$, with $M_W = gv/2$, $M_z = \frac{1}{2}v\sqrt{g^2+g'^2} = M_W/\cos\theta_W$. We see that the A field does not pick up any mass, so we identify it as the photon field.

Now look at the covariant derivative in terms of the new basis:

$$\begin{aligned} D_\mu\phi &= \partial_\mu\phi + \Big[i\frac{g}{2}\sigma_+W^- + i\frac{g}{2}\sigma_-W^+ \\ &\qquad +i\frac{g}{2}(\sigma_3(\cos\theta_W Z + \sin\theta_W A) \\ &\qquad +i\frac{g}{2}\frac{\sin\theta_W}{\cos\theta_W}(\cos\theta_W A - \sin\theta_W Z))\Big]\phi \\ &= \partial_\mu\phi + \Big[i\frac{g}{2}\sigma_+W^-_\mu + i\frac{g}{2}\sigma_-W^+_\mu + ig\sin\theta_W\frac{1+\sigma_3}{2}A_\mu \\ &\qquad +i\frac{g}{2\cos\theta_W}(\sigma_3 - (1+\sigma_3)\sin^2\theta_W)Z_\mu\Big]\phi \end{aligned}$$

We see that the photon field A_μ couples only to the upper component of ϕ, and with a charge $e = g\sin\theta_W$, which is therefore the unit of electromagnetic charge e, that of a positron.

This constitutes the standard electroweak theory of the standard model except that so far we haven't introduced any of the particles that have electroweak interactions! Next time we will introduce the leptons and quarks into this model.