Physics 613 Lecture 21 April 17, 2014

QCD; Weak Interactions

The non-Abelian theory based on SU(3), thought of as color, acting separately on each species of quark, each of which comes in three colors, is the current theory of the strong interactions.

As we mentioned some time ago, one piece of early evidence for each quark coming in three colors came from consideration of quark statistics in baryons, which seemed to have wavefunctions in space, spin and flavor that were totally symmetric despite being fermions. Multiplying that wave function by a totally antisymmetric color wavefunction ϵ_{fgh} restored the correct statistics and made sure all the known baryons were white, singlets in color. Mesons were all $(r\bar{r} + g\bar{g} + b\bar{b})$, totally SU(3) invariant, so also white.

Later, the measured values of R seemed just 3 times what one would expect from the early quark model, but just what one should expect if each quark came in three colors. A similar argument can come from the weak decays. As we will discuss later, the earliest theory of beta decay used the Fermi four-fermion interaction (1933) which is highly non-renormalizable, but is was soon realized that it could be replaced with the exchange of a heavy particle, eventually pinned down to be vector and axial vector, what we now call the $W^\pm$. That theory was also non-renormalizable as originally presented, but less badly that the four-fermion point interaction, which has dimension 6. The W has a vertex with a charged lepton turning it into a neutrino, or a d quark into a u quark (originally a neutron into a proton). So W exchange explains beta decay ${}^A_ZX \to {}_{Z+1}^{\;\;A}X + e^- + \bar{\nu}_e$ or muon decay $\mu^- \to e^-\,\bar{\nu}_e\,\nu_\mu$. If the mass of the W is much greater than the momentum transfer, which is surely the case in these decays, the interaction is indistinguishable from the nonrenormalizable 4-fermion interaction at a point. The exchange of a $W^\pm$ can convert the pair of fermions at one vertex into the pair at the other. Now if there is enough energy to create hadrons made from quark-antiquark pairs, the decay rate for $e^-\,\bar{\nu}_e$ in the final state needs to compete with all the quark posibilities as well as with the $\mu^-\,\bar{\nu}_\mu$ one, so the branching ratio for τ decay (1777 MeV) into $e^-\,\bar{\nu}_e$ is $1/(2+N_c)$, where N_c is the number of $d\bar{u}$ possibilities, *i.e.* the number of colors (we have discounted $s\bar{c}$ as not having enough phase space to have a big effect.)

The π^0 meson has nothing hadronic to decay into, but it decays into two photons at a rate much higher than the weak decays of $\pi^{\pm}$. A π^0 can decay into two photons by coupling to a charged fermion loop, with the two photons also coupling to the loop. Each possible fermion j contributes $\pm Q_j^2$. Strangely the original calculation by Steinberger in 1949, considering the pion as a proton anti-neutron pair gave a factor of 1 and the right answer, though no one would believe that physics today. The u and d quarks give[1] $(2/3)^2 - (1/3)^2 = 1/3$, so the multiple of 3 due to color is essential to get the decay rate right.

There are other verifications which require more calculations using the theory itself, and indeed there are many calculations verifying QCD as a good theory of the strong interactions, even though it is hard to calculate meaningful results at low energies.

We will come back to consider QCD interactions, the rest of Chapter 14, later. First, let's say a bit more about

Weak Interactions

As I mentioned, the weak interactions were, to some extent, understood well before there was much understanding of the strong interactions which hold nuclei together, because weak interactions can be understood in lowest order perturbation theory as some form of four-fermion interaction. The crucial step was to realize that there was a fourth fermion, because originally beta decay seemed to involve only the electron being emitted from the nucleus. Once the statistics of nuclei precluded the idea that they were made of A protons and $A-Z$ electrons, which would be fermionic whenever Z was odd, and showed they were fermionic whenever A is odd, which doesn't change in beta decay, there was a paradox. Also, the energy of the electron should be fixed in each nuclear decay by the masses of the initial and final nuclear states, and that is not the case. So Pauli proposed an invisible nearly massless fermion which eventually became the neutrino. And nuclear interactions could be described by a coupling of the form $\sum_\Gamma \bar{\psi}_p \Gamma \psi_n \bar{\psi}_e \Gamma \psi_\nu$, with some collection of Γ's that would form a Lorentz invariant. The possibilities are scalar (S), pseudoscalar (P), vector (V), axial vector (A), and tensor (T). To have full Lorentz invariance, each type would have to be the same in both $\bar{\psi}\Gamma\psi$ factors, but after 1956, when Lee and Yang suggested that parity might not be conserved in weak interactions, we could have S-P or V-A

[1]The reference in the fourth edition should be to section 12.2.3.

combinations. The particular combination of Γ's could be explored by the angular distributions in decays, and it gradually became clear that vector and axial vector terms dominated. In fact, for the leptonic vertex the form is $\bar{u}_{e^-}\gamma^\mu(1-\gamma_5)u_\nu$ with equal amounts of both. This means, of course, that only the left-handed pieces of the electron and the neutrino participated in the interaction.

Let us see the consequence of this form of interaction by answering a question one of you asked earlier, why does the π^+ decay into $\mu^+\,\nu_\mu$ at 8100 times the rate of decay into $e^+\,\nu_e$? The decay amplitude is

$$\begin{aligned}\mathcal{M} \;\propto\;& p_\pi^\alpha \bar{u}_\nu(p_\nu)\gamma_\alpha(1-\gamma_5)v_\mu(p_\mu)\\ &= \bar{u}_\nu(p_\nu)(\not{p}_\nu+\not{p}_\mu)(1-\gamma_5)v_\mu(p_\mu)\\ &= \bar{u}_\nu(p_\nu)(m_\nu(1-\gamma_5)+(1+\gamma_5)\not{p}_\mu)v_\mu(p_\mu)\\ &= -\bar{u}_\nu(p_\nu)(m_\nu(1-\gamma_5)+(1+\gamma_5)m_\mu)v_\mu(p_\mu)\end{aligned}$$

In the rest frame,

$$\Gamma = \sum_{\text{spins}} \frac{1}{2m_\pi}\int \frac{d^3p_\nu}{(2\pi)^2 4E_\mu E_\nu}\delta(m_\pi - E_\mu - E_\nu)\,|\mathcal{M}|^2\,.$$

Neglecting the neutrino mass, and using $\int \frac{d^3p}{E_\nu}\delta(m_\pi - p - \sqrt{m_\mu^2+p^2}) = 4\pi\frac{pE_\mu}{m_\pi}$, and evaluating

$$\begin{aligned}\sum_{\text{spins}}|\mathcal{M}|^2 &= m_\mu^2\sum_{\text{spins}}(\bar{u}_\nu(p_\nu)(1+\gamma_5)v_\mu(p_\mu))(\bar{u}_\nu(p_\nu)(1+\gamma_5)v_\mu(p_\mu))^\dagger\\ &= m_\mu^2\sum_{\text{spins}}(\bar{u}_\nu(p_\nu)(1+\gamma_5)v_\mu(p_\mu))\bar{v}_\mu(p_\mu)(1-\gamma_5)u_\nu(p_\nu)\\ &= m_\mu^2\,\mathrm{Tr}\left[\not{p}_\nu(1+\gamma_5)(\not{p}_\mu - m_\mu)(1-\gamma_5)\right]\\ &= m_\mu^2\,\mathrm{Tr}\left[\not{p}_\nu(1+\gamma_5)(\not{p}_\mu(1-\gamma_5)\right]\\ &= m_\mu^2\,\mathrm{Tr}\left[\not{p}_\nu\not{p}_\mu(1-\gamma_5)^2\right]\\ &= 2m_\mu^2\,\mathrm{Tr}\left[\not{p}_\nu\not{p}_\mu\right] = 8m_\mu^2\,p_\mu\cdot p_\nu = 4m_\mu^2(m_\pi^2-m_\mu^2)\end{aligned}$$

As $m_\pi = |\vec{p}| + \sqrt{m_\mu^2+\vec{p}^2}$, $|\vec{p}| = \frac{m_\pi^2-m_\mu^2}{2m_\pi}$. Thus we have

$$\Gamma \propto \frac{|\vec{p}|E_\mu}{m_\pi^2 E_\mu}m_\mu^2(m_\pi^2-m_\mu^2) = m_\mu^2\frac{(m_\pi^2-m_\mu^2)^2}{2(m_\pi)^3}.$$

Of course the same calculation holds for the electron instead of the muon, and thus

$$\begin{aligned}\frac{\Gamma(\pi \to \mu\nu_\mu)}{\Gamma(m_p \to e\nu_e)} &= \left(\frac{m_\mu}{m_e}\frac{m_\pi^2 - m_\mu^2}{m_\pi^2 - m_e^2}\right)^2 = \left(\frac{105.66}{.51100}\frac{139.57^2 - 105.66^2}{139.57^2 - 0.511^2}\right)^2 \\ &= (206.77 * 8315.7/19480)^2 = 7790.\end{aligned}$$

Well, close enough.

Weak interactions as a Gauge Theory

Thus we have good evidence that the weak interactions are carried by a $V-A$ vertex for a charged massive vector particle $W^\pm$. But we know that vector particles lead to non-renormalizable field theories unless protected by Ward identities, which are available only to massless particles. Now if we ignore the fact that the weak interactions are very short range, and therefore the $W^\pm$ is nowhere near massless, we might observe that some properties seem to fit with the idea of gauge theory.

The $W^\pm$ are vectors under proper Lorentz transformations, as are gauge particles, and they seem to couple to (p, n) or (u, d) as isospin does, and we might imagine that the (ν_e, e^-) and (ν_μ, μ^-) are also isodoublets of a sort (weak isospin doublets). Furthermore, they seem to couple to the different multiplets with comparable strengths, just as the gluons couple to the different flavors with the same strength. So we might imagine that there are gauge fields W^μ that couple to the weak-isodoublet indices on the quark in a manner similar to the way the gluons couple to the color indices. But with one important difference — the gluons couple to $\bar\psi\gamma^\mu\psi$, that is, equally to both helicity states of the fermion, whereas the W's couple only to the left-handed helicity states with $\bar\psi\gamma^\mu(1-\gamma_5)\psi$. That is, from the point of view of the weak interactions, the right-handed fermions and the left-handed fermions transform as different representations of whatever the gauge symmetry group is, with the right handed ones singlets.

What should be the group? Now the weak interactions we have seen vaugely suggest SU(2), with all the weak isospin doublets we have already encountered. Actually, we will see that the right path to the standard model is SU(2)× U(1). This means there must be four gauge particles, three for SU(2) and one for U(1). U(1), of course, is Abelian, and is the gauge group of electromagnetism, so we might imagine that that will be the photon, but the SU(2) should have an isotriplet of W's, $W^\pm$ but also a neutral one. While the charged weak currents of $W^\pm$ were well known much earlier, the neutral currents a W^0 would convey were unknown until Glashow and then Salam and Weinberg came up with their model in the late '60's, and suggested they be looked for. They were found in 1973.

From the group point of view, the U(1) and the SU(2) are separate, so if things were as simple as I just hinted, we would not talk of an electroweak unified theory but rather of separate weak and electromagnetic interactions, just as we do separate strong interactions. But things are not quite that simple, and the reason is that the electroweak symmetry is a symmetry of the fundamental lagrangian but NOT a symmetry of our universe, because the symmetry is broken. So before we get further on the standard model itself, we need to talk about spontaneous symmetry breaking. This is what we will get to next week.