Physics 613 — Spring, 2014
Homework #2. — Due Feb. 10 at 4:00

Note: if you already did part 1. in homework 1, just give the answer about η and v.

1: Consider a Lorentz transformation

$$\Lambda = e^{-i\omega_{\alpha\beta}L^{(\alpha\beta)}/2}$$

with $\omega_{\alpha\beta} = -\omega_{\beta\alpha}$.

1. Show that $\omega_{0j} = \eta = -\omega_{j0}$, others 0, generates a Lorentz transformation for $\mathcal{O} \to \mathcal{O}'$ with $\mathcal{O}'$ moving to the left, or along the $-j'^{\text{th}}$ axis with constant speed v. What is the connection of η and v?

2. Show that $\omega_{jk} = \epsilon_{jk\ell}\theta$ for one ℓ, others 0, generates a fixed rotation about the ℓ'^{th} spatial direction by an angle θ.

3. Given that a positive energy electron at rest with spin up along the z axis is given by the spinor $(1,0,0,0)^T$, and given that the Lorentz generators $L^{(\alpha\beta)}$ act as $S(L^{(\alpha\beta)}) = \frac{i}{4}\left[\gamma^\alpha, \gamma^\beta\right]$, find the spinor for a positive helicity electron with momentum p in the $+z$ direction.

4. Do the same starting with a state at rest with spin down, to get the function for a negative helicity state with $\vec{p}$ in the $+z$ direction. Show that we are consistent with 3.47.

2: As we have seen, under a Lorentz transformation that maps $x \to x'$ with

$$x'^\mu = \Lambda^\mu{}_\nu x^\nu = \left(e^{-i\omega_{\alpha\beta}L^{(\alpha\beta)}/2}\right)^\mu{}_\nu x^\nu,$$

transforms a Dirac spinor $\psi \to \psi'$ with

$$\psi'_a(x') = D_{ab}(\Lambda)\psi_b(x), \quad \text{where} \quad D_{ab}(\Lambda) = \left(e^{\omega_{\alpha\beta}[\gamma^\alpha,\gamma^\beta]/8}\right)_{ab}.$$

Using the hermiticity properties (Lecture 3 page 6) of the γ matrices, show that the $\psi^\dagger \to \psi'^\dagger$ transformation is **not** $\psi'^\dagger(x') = \psi^\dagger(x)D\left(\Lambda^{-1}\right)$ as one would have expected if $D(\Lambda)$ were a unitary representation.

(a) Find the correct expression for $\psi'^\dagger(x')$, and also for the transformation of $\bar\psi := \psi^\dagger\gamma^0$.

(b) Show that the n-index tensor field

$$T^{\mu_1,\mu_2,\cdots,\mu_n}(x) := \bar\psi(x)\gamma^{\mu_1}\gamma^{\mu_2}\cdots\gamma^{\mu_n}\psi(x)$$

transforms as an n-covariant tensor field should, that is,

$$T'^{\mu_1,\mu_2,\cdots,\mu_n}(x') = \left(\prod_{j=1}^n \Lambda^{\mu_j}{}_{\nu_j}\right) T^{\nu_1,\nu_2,\cdots,\nu_n}(x).$$