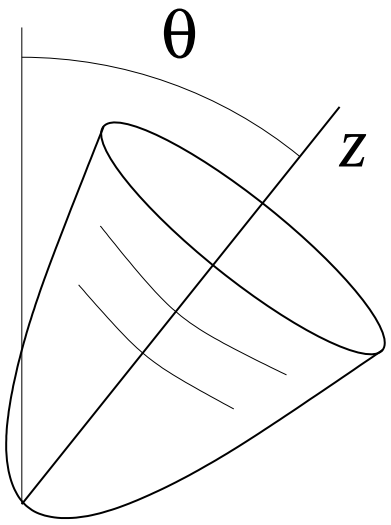
Physics 507 Homework #8
Due: Nov. 4, 2010

1. First, redo the last problem from the midterm:

8.1 Consider a paraboloid defined in polar coordinates (r, ϕ, z) by $z = \alpha r^2$, where α is a positive constant. The paraboloid is fixed in space so that its symmetry axis makes an angle of $\theta < \pi/2$ with respect to the upward vertical direction. This is in the usual gravitational field with $\vec{F} = mg$ downward.

A point particle is constrained to move without friction on the paraboloid. Being careful with the meaning of your variables,



(a) Give the Lagrangian in terms of a set of unconstrained variables.

(b) Are there any conserved quantities? What are they?

(c) There is a stable fixed point. What are its coordinates?

(d) What are the normal modes of small oscillations about the fixed point, and what are the frequencies of each of these modes?

8.2 In considering the limit of a loaded string we found that in the limit $a \to 0, n \to \infty$ with ℓ fixed, the modes with fixed integer p became a smooth excitation $y(x,t)$ with finite wavenumber k and frequency $\omega = ck$.

Now consider the limit with $q := n + 1 - p$ fixed as $n \to \infty$. Calculate the expression for y_j in that limit. This will not have a smooth limit, but there is nonetheless a sense in which it can be described by a finite wavelength. Explain what this is, and give the expression for y_j in terms of this wavelength.

8.3 Consider the Navier equation ignoring the volume force, and show that

a) a uniform elastic material can support longitudinal waves. At what speed do they travel?

b) an uniform elastic material can support transverse waves. At what speed do they travel?

c) Granite has a density of 2700 kg/m^3, a bulk modulus of 4×10^{10}N/m^2 and a shear modulus of 2.5×10^{10}N/m^2. If a short spike of transverse oscillations arrives 25 seconds after a similar burst of longitudinal oscillations, how far away was the explosion that caused these waves?