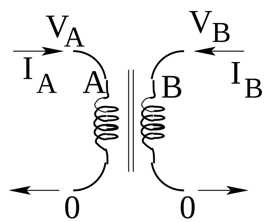
Physics 507 Homework #4

Due: Thursday, Sept. 30, 2010

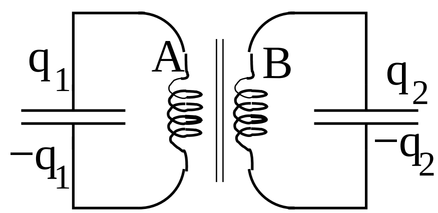
4.1 A transformer consists of two coils of conductor each of which has an inductance, but which also have a coupling, or mutual inductance.

If the current flowing into the upper posts of coils A and B are $I_A(t)$ and $I_B(t)$ respectively, the voltage difference or EMF across each coil is V_A and V_B respectively, where



$$\begin{aligned} V_A &= L_A \frac{dI_A}{dt} + M\frac{dI_B}{dt} \\ V_B &= L_B \frac{dI_B}{dt} + M\frac{dI_A}{dt} \end{aligned}$$

Consider the circuit shown, two capacitors coupled by a such a transformer, where the capacitances are C_A and C_B respectively, with the charges $q_1(t)$ and $q_2(t)$ serving as the generalized coordinates for this problem. Write down the two second order differential equations of "motion" for $q_1(t)$ and $q_2(t)$, and write a Lagrangian for this system.



4.2 A space ship is in circular orbit at radius R and speed v_1, with the period of revolution τ_1. The crew wishes to go to planet X, which is in a circular orbit of radius $2R$, and to revolve around the Sun staying near planet X. They propose to do this by firing two blasts, one putting them in an orbit with perigee R and apogee $2R$, and the second, when near X, to change their velocity so they will have the same speed as X.

- (a) By how much must the first blast change their velocity? Express your answer in terms of v_1.
- (b) How long will it take until they reach the apogee? Express your answer in terms of τ_1
- (c) By how much must the second blast change their speed? Will they need to slow down or speed up, relative to the sun.

4.3 For the Kepler problem we have the relative position tracing out an ellipse. What is the curve traced out by the momentum in momentum space? Show that it is a circle centered at $\vec{L} \times \vec{A}/L^2$, where $\vec{L}$ and $\vec{A}$ are the angular momentum and Runge-Lenz vectors respectively.

4.4 The Rutherford cross section implies all incident projectiles will be scattered and emerge at some angle θ, but a real planet has a finite radius, and a projectile that hits the surface is likely to be captured rather than scattered.

What is the capture cross section for an airless planet of radius R and mass M for a projectile with a speed v_0? How is the scattering differential cross section modified from the Rutherford prediction?