Physics 507 Homework #1
Due: Thursday, Sept. 9, 2010

- 1.[5 pt.] (a) Find the potential energy function $U(\vec{r})$ for a particle in the gravitational field of the Earth, for which the force law is $\vec{F}(\vec{r}) = -GM_E m\vec{r}/r^3$.
  (b) Find the escape velocity from the Earth, that is, the minimum velocity a particle near the surface can have for which it is possible that the particle will eventually coast to arbitrarily large distances without being acted upon by any force other than gravity. The Earth has a mass of $M_E = 6.0 \times 10^{24}$ kg and a radius of $R_E = 6.4 \times 10^6$ m. Newton's gravitational constant is $G = 6.67 \times 10^{-11}\mathrm{N} \cdot \mathrm{m}^2/\mathrm{kg}^2$.

- 2.[10 pt.] In the discussion of a system of particles, it is important that the particles included in the system remain the same. There are some situations in which we wish to focus our attention on a set of particles which changes with time, such as a rocket ship which is emitting gas continuously. The equation of motion for such a problem may be derived by considering an infinitesimal time interval, $[t, t+\Delta t]$, and choosing the system to be the rocket with the fuel still in it at time t, so that at time $t + \Delta t$ the system consists of the rocket with its remaining fuel *and also* the small amount of fuel emitted during the infinitesimal time interval.

  Let $M(t)$ be the mass of the rocket and remaining fuel at time t, assume that the fuel is emitted with velocity $\vec{u}$ with respect to the rocket, and call the velocity of the rocket $\vec{v}(t)$ in an inertial coordinate system. If the external force on the rocket is $\vec{F}(t)$ and the external force on the infinitesimal amount of exhaust is infinitesimal, the fact that $F(t)$ is the rate of change of the total momentum gives the equation of motion for the rocket.
  (a) Show that this equation is

  $$M\frac{d\vec{v}}{dt} = \vec{F}(t) + \vec{u}\frac{dM}{dt}.$$

  (b) Suppose the rocket is in a constant gravitational field $\vec{F} = -Mg\hat{e}_z$ for the period during which it is burning fuel, and that it is fired straight up with constant exhaust velocity ($\vec{u} = -u\hat{e}_z$), starting from rest. Find $v(t)$ in terms of t and $M(t)$.
  (c) Find the maximum fraction of the initial mass of the rocket which can escape the Earth's gravitational field if $u = 2000$m/s.