Physics 504 non-Homework #4
Not assigned, but good to look at

1. This is my attempt to clarify what Jackson problem 9.1 is getting at.

The development of §9.1 assumed the density and current *at each point* varies sinusoidally with time, which is not a very common occurance. As the equations that determine the fields, $\vec{A}$ in particular, are linear, it would follow that $\vec{A}(\vec{x})$ varies sinusoidally with time at each point as well. But we saw that $\vec{A}(\vec{x})$ is expanded in terms of normal modes, with each mode dependent only on one or a few multipole moments, and not the detailed configuration of the charges, so we might expect that we don't really need Eq. 9.1.

(a) Show that if one fourier transforms the vector potential,

$$\vec{A}(\vec{x},\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} dt\,\vec{A}(\vec{x},t)e^{i\omega t},$$

then

$$\vec{A}(\vec{x},\omega) = \frac{\mu_0}{4\pi}\int d^3x'\,\vec{J}(\vec{x}\,',\omega)\frac{e^{ik|\vec{x}-\vec{x}\,'|}}{|\vec{x}-\vec{x}\,'|},$$

where

$$\vec{J}(\vec{x}\,',\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} dt\,\vec{J}(\vec{x}\,',t)e^{i\omega t},$$

without any assumptions on the periodic nature of $\vec{J}$. Show this starting with the Green's function (Eq. 9.2) for the full time dependent problem.

(b) Consider a single charge q rotating about the z axis in the $x-y$ plane in a circle of radius R with angular speed ω_0. Calculate the $\ell = 0$ and $\ell = 1$ terms in the expansion for $\vec{A}$, for all frequencies. Are there any higher values of ℓ which are nonzero? For what frequencies?

2. Do problem 9.7 from Jackson. The dipole or quadripole can be assumed to be concentrated at the origin.