Physics 504 Ordinary Homework #6
Due: March 31, 2011

1. We have derived that if $\mathcal{O}'$ is moving in the x direction with velocity $c\beta$ relative to $\mathcal{O}$, the relation between the two observers' coordinates of an event are related by

$$\begin{aligned} x'^0 &= \gamma\left(x^0 - \beta x^1\right), & x'^2 &= x^2 \\ x'^1 &= \gamma\left(x^1 - \beta x^0\right), & x'^3 &= x^3, \end{aligned}$$

where we have assumed the axes are parallel.

(a) If instead the velocity $c\vec{\beta}$ of $\mathcal{O}'$ with respect to $\mathcal{O}$ is in an arbitrary direction, by rotational invariance it is clear that

$$x'^0 = \gamma\left(x^0 - \vec{\beta}\cdot\vec{x}\right),$$

and it is also clear that the components of $\vec{x}\,'$ and $\vec{x}$ parallel to $\vec{\beta}$ transform as x^1 did. This is not enough to completely determine $A^\mu{}_\nu$, but if we also impose that the spatial coordinates perpendicular to $\vec{\beta}$ are unchanged, the relationship is uniquely defined. Show that the result is Jackson's second equation 11.19.

(b) From this, derive the expression for $\vec{v}\,' = d\vec{x}\,'/dt' = c\,d\vec{x}\,'/dx'^0$ in terms of $\vec{v} = d\vec{x}/dt$ and $\vec{\beta}$, and show that the components parallel and perpendicular to $\vec{\beta}$ are given by Jackson's 11.31, with some change of notation and observing that we can interchange $\mathcal{O}$ and $\mathcal{O}'$ by reversing the direction of $\vec{\beta}$.

(c) Use the result of (a) to show that the lorentz transformation is given by $A^\mu{}_\nu$ shown in Jackson equation 11.98.

2. Do Jackson problem 11.8a