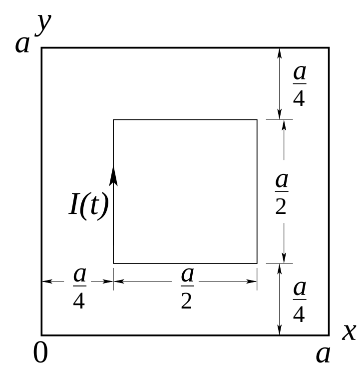
Physics 504 Ordinary Homework #5
Due: March 24, 2011

This homework asks you to redo, carefully and fully explained, the first two questions from the midterm exam.

1. Consider a square wave guide along the z direction ($-\infty < z < \infty$) with cross section $0 \le x \le a$, $0 \le y \le a$, with walls of high conductivity σ.

There is a small square loop of wire in the $z = 0$ plane, sides of length $a/2$, centered at $x = y = a/2$, the center of the wave guide, in which there is an oscillating current $I(t) = I_0 \cos \omega t$, that is, the current at $(a/4, a/2, 0)$ is $I(t)\hat{e}_y$.



(a) Which form of waves (TEM, TE, TM) will be excited in this wave guide?

(b) In the limit $\sigma \to \infty$, what will the steady state electric field $\vec{E}(x, y, z)$ be in the wave guide? The answer need not be in closed form, but should be explicit.
Do your calculations here agree with your observations of part (a)?

(c) Describe qualitatively in what ways the field on a cross section at finite positive z would depend on z and other parameters, in particular frequency.

(d) Describe qualitatively how the answer to (c) would change if the conductivity were large but not infinite.

2. We extensively considered what happens when a plane wave in a homogeneous isotropic nonmagnetic nonconducting medium hits an interface with a second such medium with a different index of refraction.

Now consider what happens if the second medium is instead a good nonmagnetic conductor, with conductivity σ and $\mu = \mu_0$. Let the incident wave make an angle θ_i with the normal to the surface, and one should consider separately the two polarizations $E_\parallel$ and $E_\perp$ as we did for the two dielectric interface. I only ask you to consider the $E_\perp$ case.

You may assume the first medium (dielectric) is vacuum.

(a) Give the forms of the $\vec{E}$ and $\vec{B}$ fields in the dielectric and the boundary conditions they satisfy.

(b) Show that the angle of reflection equals the angle of incidence.

(c) Use the boundary conditions to determine the amplitudes of the reflected fields and the reflection coefficient giving the fraction of the incident flux which is reflected.