# Intro to Lecture 21 Nov. 18, 2016

Last time we motivated the generating function for the Legendre Polynomials by looking at the potential of a point charge at $\vec{r} = (0, 0, a)$, in a power series in $a/r$. We discussed multipole moments (for axially symmetric configurations only). From the generating function, we found the recursion and derivative equations from which we found Legendre's equation, Rodriguez' formula for $P_n(z)$, and the orthogonality relations. We did a simple example of an uncharged conducting sphere in a asymptotically uniform $\vec{E}$ field.

Today:

Any axially symmetric electrostatic configuration can be treated this way, and we will first consider a ring of charge. For non-axially-symmetric configurations we will need the *associated* Legendre polynomials, but first we will discuss $P_\nu$ for $\nu \notin \mathbb{Z}$. We will have some fun wandering in the complex plane.

Then we recall that in separating variables, the $\Phi(\phi)$ gives a separation constant $m$ which we took to be zero last time but we should now take to be an arbitrary integer. The solutions no longer fit in our framework of orthogonal polynomials, but they are in fact derivatives of the Legendre polynomials. We will discuss their orthogonality properties and the more appropriate description of angular dependence in terms of the spherical harmonics $Y_\ell^m(\theta, \phi)$.

Finally we will finish this chapter by considering other classical orthogonal polynomials. These include the Hermite polynomials which are orthogonal with a gaussian weight function and give the eigenstates of a harmonic oscillator. We also have the Laguerre and generalized Laguerre equations, which give the eigenstates for hydrogenic atoms, and finally we mention some other classics, Chebyshev and the various hypergeometric functions

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- Next week contains Thanksgiving and a change in class schedule. On Wednesday, Nov. 23, we have class on the Friday schedule, at 1:40.
- Homework 9 is due Nov. 23 at the beginning of class. It consists of one question for you to answer individually, and the project discussing Helmholtz' equation in four-dimensional Euclidean space. This is to be done in two groups, each of which will submit a beautifully written report **at the start of class** at 1:40 on Nov. 23.

  The two groups are:

  - Hamzy Chaudhry, Ross Ersteniuk, Vlad Mirkovic
  - Yuanwen Dong, XinYuan Lai, with possible contributions from Phillip Rechani.

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