

Intro to Lecture 15

Oct. 28, 2016

Last time we defined the Beta function. When the sum of the arguments is an integer, this is the integral over a contour around a cut. By deforming the cut to infinity, we found a finite expression. When the sum is 1, this gives the Euler reflection formula, which will be an important relation for Gamma functions.

We then discussed the Mittag-Leffler expansion that gives the expression for a function which is analytic in the entire complex plane except for a set of isolated simple poles, and is also bounded by a power of $|z|$ in all directions as $z \rightarrow \infty$. The function is then given as a sum of simple poles $r_k/(z - z_k)$ plus a polynomial. We used this to derive the infinite product expression for $\sin x$.

Then we began the discussion of the steepest descents method of finding the asymptotic behavior of functions $I(s)$ of a real variable s defined by an integral over a contour $\int_C dz g(z) e^{sf(z)}$, based on the idea that the dominant contribution comes from the point at which $\operatorname{Re} f$ is a maximum. We found the general form and applied it to the Hankel function.

Today

We ran out of time before doing the same for Stirling's approximation for the factorial (or Gamma) function, which we will begin with today.

Then we will return to the question of the important partial differential equations of physics, involving the Laplace operator ∇^2 . We have already discussed the general idea of separating variables, and have found the equations in cartesian, spherical, and circular cylindrical coordinates. We will begin our analysis of these ordinary differential equations, but we will first focus on methods of solving a more general set of ordinary second order linear equations which emerge. We begin by classifying ordinary, regular singular and essential singular points x , and by looking for expansions of the solutions about one point. This gives us Frobenius' method for finding at least one solution expanding about an ordinary or regular singular point, and we will find the other solution, when Frobenius doesn't give us that also, by defining and using the Wronskian.

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- Homework #6 is due Monday at 5:00 PM as usual.
 - Exam solutions have been posted since the day after the exam. You should be reviewing this to learn how you might have approached things better. Please ask if there is stuff there you don't understand.
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