Intro to Lecture 12 Oct. 14, 2016

Last time we discussed infinite products and found one for $\sin x$, which we also used to get the series for $x \cot x$ and show the connection between B_{2n} and $\zeta(2n)$. Then we defined the very important Gamma function and also the incomplete Gamma function, leading into the idea of an asymptotic expansions, in particular of $\Gamma(a, x)$.

Then we just touched on the beginning of complex variables, the incredible power of adding $i = \sqrt{-1}$ to our field. This enables us to treat a two dimensional vector space (x, y) as a single complex "number" $z = x + iy$, with vector addition and scalar multiplication by reals defining addition of complex numbers and their multiplication by a real number. But if we define $i^2 = -1$, we can define multiplication of arbitrary complex numbers (commutatively), and we have defined the field $\mathbb{C}$.

Today we define complex conjugation, absolute value, and discover the connection of exponentials and trig functions. But the real power of complex variables comes when we consider analytic functions, imposing the Cauchy-Riemann conditions which turns a map $\mathbb{R}^2 \to \mathbb{R}^2$ into an analytic map $\mathbb{C} \to \mathbb{C}$. We will find that contour integrals then become immensely powerful tools, due to the Cauchy integral theorem, which requires the vanishing of an the integral around a closed path inside of which the function is analytic, and the Cauchy integral formula, which gives the value of an analytic function anywhere within such a curve by its values on the curve. And its derivatives as well. Finally we will learn how a Taylor series expansion always converges with a circle up to the nearest singularity.

- Project 1: Was due yesterday, but one group is late, and has until Monday at 5 PM to submit, with a 20% penalty. I will post solutions at that time.
- We have a midterm next Wednesday, Oct. 19. You are allowed two letter-sized sheets of paper with handwritten notes on them. It will cover everything through Lecture G, which is to say up to, but not including, today's lecture.
- Homework 5 (project was #4) is due Oct. 24.