Intro to Lecture 11 Oct. 12, 2016

Last time we used a power series expansion in the maximum angle a pendulum swings, or more accurately in $m := \sin^2\left(\frac{\theta_M}{2}\right)$, to find the period. We defined the answer to give the *complete elliptic integral of the first kind*, and found the explicit expression for the coefficients in the power series in m. We also defined the second kind, and the *Jacobi elliptic functions* and *Theta functions*.

Power series expansions of a known function, called the *generating function* can define the sequence of the coefficients as an interesting set. This gave us the Bernoulli numbers B_n and the Bernoulli functions $B_n(s)$, which we found can give improvements on the trapezoid rule for numerical integration.

We also defined the *Riemann zeta function* $\zeta(x)$ and found its values at positive even integers are related to the Bernoulli number.

Today we will discuss a bit more on the Riemann zeta function, including one entering the energy density of a black box, and then leading to discussion of infinite products. We will find one for $\sin x$, which we will use to justify the series for $x \cot x$ and to show the connection between B_{2n} and $\zeta(2n)$.

One of the most important special functions is the Gamma function $\Gamma(x)$ which generalizes the factorial. We will begin with the more general incomplete Gamma function $\Gamma(a, x)$. After specializing to $\Gamma(x)$ we will discuss *asymptotic series*, in particular for $\Gamma(a, x)$ for large x. Finally we get one more function, the *logarithmic integral function*.

Next time on to complex variables.

- Project 1: It is due tomorrow. What are the groups? Before last lecture I had

  A Zongjie, XinYuan, Phillip

  B Ross, Vlad, Abdul

  C Yuanwen, James, Hamza

  but it would be better if one of B moved to C.

  Each group should submit **one** report, beautifully explained and typeset, by 5 PM.

- We will have a midterm on Wednesday Oct. 19. You are allowed two letter-sized sheets of paper with handwritten notes on them. It will cover everything through Lecture G, which is to say through today's lecture.

- Homework 5 (project was #4) is due Oct. 24.