Intro to Lecture 8 Sept 30, 2016

Last time we discussed integration of n-forms and Stokes' theorem. Then we talked about Hodge duality, which means in three dimensions that vectors are associated with 1-forms or 2-forms, and 3-forms are associated with scalars (or, some would say, scalar densities). Here we defined the covariant volume 3-form and the covariant ε.

Then we found the general expression for the Laplacian.

Then we discussed limiting ourselves to orthogonal coordinates for Euclidean space, with vectors described by orthonormal basis vectors aligned with the (non-Cartesian) coordinates. In terms of these orthogonal coordinate systems we derived the gradient of a scalar, the curl and the divergence of a vector field, and finally the laplacian of a scalar field. Finishing the discussion of differential operators in orthogonal coordinates will enable us to investigate many problems involving partial differential equations in physics.

We will begin today with a diversion to discuss how forms enlighten our relativistic understanding of electromagnetism. After discussing the Minkowski metric which describes space-time in special relativity, we will discuss Maxwell's Equations in terms n-forms in 4 dimensional Minkowski space.

But then we will turn to the solution of the most important linear partial differential equations of physics. In particular, we will see how to use separation of variables in several contexts. We will meet a number of famous equations.

- Homework 3 is due Monday
- Homework 4 will be a "Project", to be worked on in groups. I asked you on Wednesday to form groups, so let's get that down now. Get together with your group after class today just to exchange contact information and discuss how to proceed.

  The project is going to involve finding the electrostatic field of charged ellipsoidal conductors in otherwise empty space. That means solving for the electrostatic potential as a solution of laplace's equation, taking a constant nonzero value on the ellipsoid and going to zero at infinity. Then you will evaluate the capacitance (the charge divided by the potential) of a thin conducting disk of radius R and of a straight thin wire of length L.
- I originally tentatively scheduled the midterm for Oct. 12, because I was under the impression that warning grades were due by the 14'th. But in fact we have one more week. Shall we delay the midterm to Oct. 19?