Intro to Lecture 3 (Sept 14, 2016)

On Friday we discussed using non-cartesian coordinate systems and choosing basis vectors appropriately, which means they differ at different points in space. We discussed that the derivative of a vector field involves two directions, so in three dimensions there are 9 partial derivatives involved. We discussed one particular combination, the divergence of a vector field, and what the interpretation of the divergence is, physically. We saw how for cylindrical coordinates the divergence takes a form slightly different from that in Cartesian coordinates.

For today, the topic will be the other vector derivatives and notions of integration, which we will consider in cartesian coordinates. In a week's time we will return to discuss these differential and integral concepts much more generally, both in generalized coordinates for E^3 and then even when the underlying space is not flat E^3.

But for today, we will stick to Cartesian coordinates. Last time, after defining the gradient and the divergence, we defined the cross-product of vectors, which leads us to defining the curl of a vector field, and then the laplacian of a scalar.

Finally we will discuss integration, in particular of vector fields along curves and over surfaces, as well as volume integrals. We will see that Stokes' theorem and Gauss' theorem are just vector-field versions of the fundamental theorem of calculus, and give applications in physics. Finally we will discuss Helmholtz' theorem, permitting the decomposition of a vector field into irrotational and solenoidal parts.

Probably not until next time, we will go more into vector spaces, discussing more formally linear transformations, inner products and norms, dual space, and morphisms. Then we will be ready, probably next Wednesday, to begin our discussion of manifolds and differential geometry.

Remember: Homework 1 due this coming Monday, Sept. 19, at 5:00.

[Remind students (especially new ones) of my contact information, web pages, and that they should talk to each other.]

So last time we were looking at the 9 components of the partial derivatives of a vector field, $\dfrac{\partial}{\partial x_i}V_j$, and we did thoroughly discuss one part of this, the divergence,

$$\vec{\nabla} \cdot \vec{V} = \sum_i \frac{\partial}{\partial x_i} V_i.$$