Physics 464/511 Homework #8
Due: Nov. 14, 2015 at 5:00 P. M.

1 [10 pts] **(a)** [6 pts] Show that the first derivatives of the Legendre polynomials satisfy a self-adjoint differential equation with eigenvalue $\lambda = n(n+1) - 2$.
(b) [4 pts] Show that these Legendre polynomial derivatives satisfy an orthogonality relation

$$\int_{-1}^{1} P_m'(x) P_n'(x)(1-x^2)\, dx = 0 \quad \text{for } m \neq n.$$

2 [5 pts] In a Maxwellian distribution the fraction of particles between the speed v and $v + dv$ is

$$\frac{dN}{N} = 4\pi \left(\frac{m}{2\pi kT}\right)^{3/2} \exp\left(\frac{-mv^2}{2kT}\right) v^2\, dv,$$

N being the total number of particles. The average or expectation value of v^n is defined as $\langle v^n \rangle = N^{-1} \int v^n dN$. Show that

$$\langle v^n \rangle = \left(\frac{2kT}{m}\right)^{n/2} \left(\frac{n+1}{2}\right)! \Big/ \left(\frac{1}{2}\right)! \,.$$

3 [8 pts] Verify the contour integral representation of $\zeta(s)$,

$$\zeta(s) = -\frac{(-s)!}{2\pi i} \int_C \frac{(-z)^{s-1}}{e^z - 1} dz$$

where C comes in above the real axis at $+\infty$, passes the origin counter-clockwise, and leaves to $+\infty$ below the cut along real $z \geq 0$. The points $z = \pm 2n\pi i, n \in Z^+$ are excluded

4 [10 pts] Show, using the results of the last problem, that $\zeta(s)$ is analytic in the entire finite complex plane except at $s = 1$ where it has a simple pole of residue $+1$.