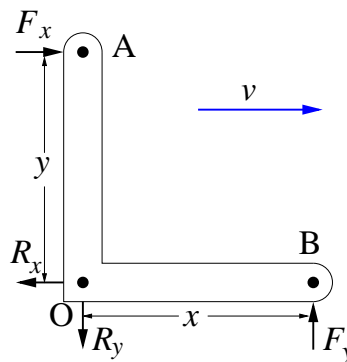


Due: Dec. 12, 2016 at 5:00 PM

1 Consider a rigid bracket, a carpenter’s right angle “framing square” as shown. At the vertex at the origin O , a force with components $(-R_x, -R_y)$ acts, and at the ends, at A and B , forces F_x and F_y act, horizontally and vertically, respectively. In the rest frame of the bracket, the magnitudes of R_x , R_y , F_x and F_y are all equal (call that F) and the sides are also equal of length L , so $x = y = L$, so there is no net force and no net torque on the bracket, and it remains at rest.



Now consider this situation as viewed by an observer \mathcal{O}' for whom the bracket is moving to the right with uniform velocity v .

- (a) Describe the situation as seen by \mathcal{O}' , including the magnitudes and directions of each of the forces and the distances between the points O and A and between O and B . Note that it is the 4-force $f^\mu = dP^\mu/d\tau$ on a particle which transforms as 4-vector, not dP^μ/dt .
- (b) What are the total force and total torque on the bracket according to \mathcal{O}' .
- (c) Is the bracket accelerating, and is it rotating, according to \mathcal{O}' ?
- (d) What is happening to the angular momentum of the bracket, according to \mathcal{O}' ?

If you have answered these questions correctly, the results should present a paradox. That is, they should not be in accord with naïve notions.

- (e) Explain how the results can be reconciled with notions that torque gives the rate of change of angular momentum and that angular momentum is a covariant quantity that should transform suitably between different observers.

Hint: the force at A is doing positive work on the bracket, and the force R'_x is doing negative work at O . You will need to consider the energy-momentum tensor $T^{\mu\nu}(x^\alpha)$ as well as how conservation affects the total 4-momentum P^μ and the total angular momentum of the bracket.

Clearly this is a very tricky question. Those of you with poor backgrounds in special relativity will probably want to cooperate with some students with more experience. Give it a good try!