Physics 464/511 Homework #1

Due: Sept. 19, 2016 at 5:00 P.M.

1 [5 pts] Using the Levi-Civita symbol ϵ, show that

$$\vec{A} \times \left(\vec{B} \times \vec{C}\right) = \vec{B}\left(\vec{A} \cdot \vec{C}\right) - \vec{C}\left(\vec{A} \cdot \vec{B}\right).$$

2 [5 pts] Consider a parallelopiped whose edges at one vertex are given by three vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$. Show that it has a volume given by $\vec{A} \cdot (\vec{B} \times \vec{C})$. Note that the volume is the area of one face times the distance (measured perpendicular to that face) between that face and the one opposite.

3 [5 pts] Show that, if $\vec{u}$ and $\vec{v}$ are irrotational fields (*i.e.* $\vec{\nabla} \times \vec{u} = 0 = \vec{\nabla} \times \vec{v}$) then $\vec{u} \times \vec{v}$ is solenoidal (*i.e.* $\vec{\nabla} \cdot (\vec{u} \times \vec{v}) = 0$).

4 [5 pts] Work out the Leibnitz rule for the curl of a product of a scalar field and a vector field; that is, express $\vec{\nabla} \times (\rho\vec{v})$ in terms of ρ, $\vec{\nabla}\rho$, $\vec{v}$, and $\vec{\nabla} \times \vec{v}$.

5 [5 pts] Show that any solution of the equation

$$\vec{\nabla} \times \vec{\nabla} \times \vec{A} - k^2\vec{A} = 0$$

with $k \neq 0$ automatically satisfies the vector Helmholtz equation

$$\nabla^2\vec{A} + k^2\vec{A} = 0$$

and the solenoidal condition

$$\vec{\nabla} \cdot \vec{A} = 0.$$