PHYSICS 617 Spring 2022

GENERAL RELATIVITY
(Tentative Syllabus)

This is a tentative schedule of what we will cover in the course. It is subject to change, often without notice. These will occur in response to the speed with which we cover material, individual class interests, and possible changes in the topics covered. Use this plan to read ahead from the textbooks,

[1] Sean Carrol, "Spacetime and Geometry".

[2] D. Lovelock and H. Rund, "Tensors, Differential Forms, and Variational Principles", Dover Publications, Inc, New York

[3] H.Goldstein, C. Poole and J. Safko, "Classical Mechanics", Third edition, Addison Wesley

so you are better equipped to ask questions in class.

- **PRELIMINARIES**

  Space-Time in Classical Physics: Euclidean structure. Causal structure. Inertial frames. Gravity vs Inertia. Equivalence Principle.

  Suggested literature: Lecture notes
  Secs. 1.1-1.3, 2.1-2.3 in [2]
  Secs. 4.1-4.8 in [3] (kinematics of rigid body motion)

  Space-Time in Special Relativity: Causal structure in SR. Poincaré group. Pseudo-Euclidean (Mikowskian) space. Vectors and tensors.

  Suggested literature: Lecture notes
  Secs. 1.1-1.7 in [1]
  Secs.7.1-7.5 in [3] (classical mechanics of special theory of relativity)

  Basic facts from CM and EM (self study): Maxwell's equations. Energy and momentum. Classical field theory.

  Suggested literature: Secs.1.8-1.10 in [1]
  Secs. 13.1-13.7 in [3] (Lagrangian formulations for continuous systems and fields)

  Curvilinear coordinates: Equivalence Principle and Special Relativity. Transformations of coordinates. Vectors and tensor fields. Parallel vector fields. Covariant (absolute) differential. Covariant derivative. Christoffel symbols. Ellipsoidal coordinates in $\mathbb{R}^3$.

  Suggested literature: Lecture notes
  Secs. 2.4.-2.6 in [2]

- **ELEMENTS OF THE DIFFERENTIAL GEOMETRY**

  Manifolds: Topological space. Definition of the manifold. Examples: $\mathbb{S}^n$, $O(N)$, $SO(N)$, $\mathbb{R}P^n$, $\mathbb{T}^n, \ldots$.

  Suggested literature: Lecture notes
  Secs. 2.1, 2.2 in [1]
  Sec. 3.1 in [2]

  Tensor fields on manifolds: Tangent vector and tangent space. $T_{\rm P}M$ as space of derivations. Tangent vector fields and tangent bundle. Cotangent space and cotangent fields. Tensor fields. Tensor densities. Levi-Civita symbols.

  Suggested literature: Lecture notes
  Secs. 2.3, 2.4, 2.8 in [1]
  Secs. 3.2, 3.3, 4.1, 4.2 in [2]

  Affinely connected manifolds: Absolute differential and covariant derivative. Affine connection. Torsion. Parallel transport. Geodesics. Parallel transport along closed curves. Curvature (Riemann) tensor.

  Suggested literature: Lecture notes
  Secs. 3.3-3.7 in [2]

  Basic facts on differential forms (self study): Differential form. Exterior derivative. Closed and exact forms. Wedge product. Integration. Stokes's theorem. Hodge star operator.

  Suggested literature: Secs. 2.9, 2.10, Appendix E [1]
  Secs. 5.1-5.3, 5.5 in [2]

- **SPACE - TIME IN GENERAL RELATIVITY**

  (pseudo-)Riemannian manifolds: Metric. Physical coordinates. Geodesics in a (pseudo-)Riemannian manifold. Locally geodesic coordinates.

  Suggested literature: Lecture notes
  Secs. 2.5, 3.1-3.4 in [1]
  Secs. 7.1, 7.2 in [2]

  Levi-Civita connection: Particle in the gravitational field (Free motion. Newtonian limit). Absence of torsion in General Relativity. Connection vs metric.

  Suggested literature: Lecture notes
  Secs. 3.1-3.3, in [1]

  Curvature tensor in (pseudo) Riemannian space: Curvature vs metric. Flatness condition. Properties of the Riemann tensor: Symmetries, number of independent components, Bianchi identity. Free fall and Fermi normal coordinates (self study). Geodesic deviation equation.

  Suggested literature: Lecture notes
  Secs. 3.6, 3.7, 3.10 in [1]
  Secs. 7.3 in [2]
  For Fermi normal coordinates, see F.K. Manasse and C.W. Misner, "Fermi normal coordinates and some basic concepts in differential geometry", Journal of Mathematical Physics **4**, no. 6, p. 735 (1963)
  For physical meaning of the geodesic deviation equation, see sec.1.6 in iC.W. Misner, K.S. Thorne, J.A. Wheeler, "Gravitation".

- **GRAVITATIONAL FIELD EQUATIONS**

  Einstein equations: Energy-Momentum tensor. Einstein's equation. Coordinate conditions. Harmonic coordinates.

  Suggested literature: Lecture notes
  Secs. 4.1, 4.2 in [1]

  Weak gravitational field: Linear approximation. Non-relativistic matter. Propagation of light in a weak gravitational field. Hamilton-Jacobi method in Classical Mechanics. Frequency shift in weak gravitational field. Deflection of light ray in the gravitational field of Sun. Gravitational lenses.

  Suggested literature: Lecture notes
  Secs. 10.1-10.5 in [3] (Hamilton-Jacobi theory)
  Secs. 7.1-7.3 in [1]

  Variational principle: Lagrangian formulation. Derivation of Einstein's equation from variational principle.

  Suggested literature: Lecture notes
  Secs. 1.10, 4.3-4.5 in [1]
  Secs. 8.1-8.5 in [2]

- **EXACT SOLUTIONS OF EINSTEIN'S EQUATION**

  Schwarzschild solution: Static and stationary solutions. Rotationally invariant metrics. Schwarzschild metric.

  Suggested literature: Lecture notes
  Secs. 5.1, 5.3 in [1]

  Rotational symmetry and relativistic stellar structure: Gravitating spherical body. Tolman-Oppenheimer-Volkoff equation. Incompressible matter. Gravitational stability condition for a static star. Buchdahl's theorem.

  Suggested literature: Lecture notes
  Sec. 5.8 in [1]

  Geodesics of Schwarzschild: Kepler problem. Fall to the center.

  Suggested literature: Lecture notes
  Secs. 5.4-5.6 in [1]
  Secs. 3.7, 3.8 in [3] (Kepler in Classical Mechanics problem)

  The maximally extended Schwarzschild solution: Rindler space-time. Kruskal coordinates.

  Suggested literature: Lecture notes
  Sec. 5.7 in [1]

  Isometries: Killing vectors. Lie bracket. Birkhoff's theorem.

  Suggested literature: Lecture notes
  Secs. 3.8, 5.2, Appendix B in [1]
  Sec. 4.4 in [2]

  Gravitational collapse: Comoving coordinates. Synchronous coordinates (Gaussian normal coordinates). Collapse of dust-like matter.

  Suggested literature: Lecture notes
  Secs. 11.8, 11.9 in S. Weinberg, "Gravitation and Cosmology: Principles and Applications of The General Theory of Relativity", John Wiley & Sons, Inc.
  Secs. 7.2, Appendix D in [1]

- **GRAVITATIONAL RADIATION**

  Gravitational waves: Gravitational wave solutions. Production of gravitational waves.:w

  Suggested literature: Lecture notes
  Secs. 7.4, 7.5 in [1]

  Energy-momentum of gravitational field: Gravitational energy-momentum pseudo-tensor. Gravitational energy-momentum in linear approximation.

  Suggested literature: Lecture notes

  Energy loss due to gravitational radiation: Energy loss. Quadrupole radiation. Gravitational radiation of a double star.

  Suggested literature: Lecture notes
  Secs. 7.6, 7.7 in [1]

- **UNIFORM ROTATION**

  Rotating star: Weak field approximation for uniform rotation. Orbit precession due to rotation of central body. Kerr metric.

  Suggested literature: Lecture notes
  Secs. 6.1, 6.6 in [1]
  Secs. 3.7-3.9 in [3] (Kepler in Classical Mechanics problem, Laplace-Runge-Lenz vector)

  J. Lense, H. Thirring, "*On the influence of the proper rotation of a central body on the motion of the planets and the moon, according to Einstein's theory of gravitation*", Phys. Z.,**19**, 156 (1918)

- **GENERAL RELATIVITY AND COSMOLOGY**

  Geometry of isotropic spaces: Isotropic and homogenous manifolds. Spatially isotropic Space-Time

  Suggested literature: Lecture notes
  Sec. 8.1 in [1]

  Cosmological models: Robertson-Walker metric. Friedman equations. Role of the cosmological constant. Hubble's law. Big Bang. FRW cosmology. Vacuum-dominated universe: de Sitter model.

  Suggested literature: Lecture notes
  Secs. 8.2-8.4 in [1]

  Isotropic models and observations (self study): Cosmological redshift and distances. Our universe. Inflation.

  Suggested literature: Secs. 8.5-8.8 in [1]