PHYSICS 611 Spring 2020

STATISTICAL MECHANICS

TENTATIVE SYLLABUS

This is a tentative schedule of what we will cover in the course. It is subject to change, often without notice. These will occur in response to the speed with which we cover material, individual class interests, and possible changes in the topics covered. Use this plan to read ahead from the text books, so you are better equipped to ask questions in class. I would also highly recommend you to watch Prof. Kardar's lectures online at

https://ocw.mit.edu/courses/physics/8-333-statistical-mechanics-i-statistical-mechanics -of-particles-fall-2013/index.htm"

- **PROBABILITY**

  Probability: Definitions. Examples: Buffon's needle, lucky tickets, random walk in one dimension. Saddle point method. Diffusion equation. Fick's law. Entropy production in the process of diffusion.

  One random variable: General definitions: the cumulative probability function, the Probability Density Function (PDF), the mean value, the moments, the characteristic function, cumulant generating function. Examples of probability distributions: normal (Gaussian), binomial, Poisson.

  Many random variables: General definitions: the joint PDF, the conditional and unconditional PDF, expectation values. The joint Gaussian distribution. Wick's theorem. Central limit theorem.

- **ELEMENTS OF THE KINETIC THEORY OF GASES**

  Elements of Classical Mechanics: Virial theorem. Microscopic state. Phase space. Liouville's theorem. Poisson bracket.

  Statistical description of a system at equilibrium: Mixed state. The equilibrium probability density function. Basic assumptions of statistical mechanics.

  Bogoliubov-Born-Green-Kirkwood-Yvon (BBGKY) hierarchy: Derivation of the BBGKY equations. Collisionless Boltzmann equation. Solution of the collisionless Boltzmann equation by the method of characteristics. Vlasov equations.

  Boltzmann equation: Length and time scales in the BBGKY hierarchy. Binary collisions. Differential cross section. Mean free path. Dilute gas approximation. Bogoliubov's condition (Boltzmann's hypothesis of molecular chaos). Bogoliubov's form of the collision integral. Boltzmann's collision integral. Heuristic "derivation" of the Boltzmann equation.

  General consequences of the Boltzmann equation: H - theorem and irreversibility. Equilibrium properties: the equilibrium distribution, the ideal gas entropy. Collision-time approximation for the Boltzmann equation.

  Chapman-Enskog method (self study): Conservation laws. Zero and first order hydrodynamics ("Statistical Physics of Particles", Ch.3.7-3.9).

- **FUNDAMENTAL PRINCIPLES OF STATISTICAL MECHANICS**

  Microcanonical ensemble in classical Statistical Mechanics: Rôle of the integrals of motion. Microcanonical distribution. The classical density of states, the number of states, the statistical weight of a macroscopic state. Ergodic hypothesis. Partial equilibrium and macroscopic states. Classical density of states for the monoatomic ideal gas and polar rods. Statistical integral. Laplace transform.

  Normal systems in Statistical Thermodynamics (I): Asymptotic forms of the number of states and state density of a macroscopic system. Entropy of normal systems. Statistical temperature. Helmholtz free energy. Legendre transformation. Mixing entropy and Gibbs paradox. The Tonks gas. Partition of energy between two systems in thermal contact.The zeroth law of thermodynamics.

  Normal systems in Statistical Thermodynamics (II): Quasi-static adiabatic processes in statistical mechanics: adiabatic theorem in classical mechanics, adiabatic invariants, particle in an infinite well. Adiabatic theorem in statistical mechanics. The first law of thermodynamics. Pressure. One dimensional gas with nearest neighbor interactions. Gibbs free energy. Enthalpy. Chemical potential. Gibbs-Duhem relation.

  Normal systems in Statistical Thermodynamics (III): Additivity of entropy for systems in equilibrium. Increase of entropy by the establishment of a new equilibrium. The second law of thermodynamics: Clausius and Kelvin formulations. Clausius theorem. Maximum work done by a body in an external medium. Stability conditions.

  Microcanonical ensemble in quantum Statistical Mechanics: Equipartition theorem. The splendors and miseries of classical Statistical Mechanics. The third law of thermodynamics. Quantum micro and macro states. The density matrix and its properties. The two-level system. Quantum microcanonical distribution. Entropy in the quantum microcanonical ensemble. Thermal density matrix and von Neumann entropy. Thermodynamics of the two level system.

- **GIBBS DISTRIBUTIONS**

  Canonical ensembles: Physical interpretation of the thermal density matrix. Energy PDF in the canonical ensemble. Gibbs canonical ensemble. Fluctuations of the fundamental thermodynamic quantities. Grand canonical ensemble. Landau free energy. The particle number fluctuations.

  Canonical examples: Dilute polyatomic gases. Vibrations of a solid: phonons, Einstein and Debye models. Black-body radiation: Stefan-Boltzmann and Wien's displacement laws.

- **IDEAL QUANTUM GASES**

  Ideal quantum gases:. Hilbert space of identical particles. Canonical and grand canonical ensembles.

  Ideal Fermi gas: Equation of state of an ideal Fermi gas. Examples: Pauli paramagnetism, white dwarf stars.

  Ideal Bose gas: The degenerate Bose gas. Bose-Einstein condensation. Superfluids ("Statistical Physics of Particles", Ch.7.7).