

Hot Dark Matter in Cosmology

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1 Historical Summary

Cosmological dark matter in the form of neutrinos with masses of up to a few electron volts is known as hot dark matter. In 1979-83, this appeared to be perhaps the most plausible dark matter candidate. Such HDM models of cosmological structure formation led to a top-down formation scenario, in which superclusters of galaxies are the first objects to form, with galaxies and clusters forming through a process of fragmentation. Such models were abandoned when it was realized that if galaxies form sufficiently early to agree with observations, their distribution would be much more inhomogeneous than it is observed to be. Since 1984, the most successful structure formation models have been those in which most of the mass in the universe is in the form of cold dark matter (CDM). But mixed models with both cold and hot dark matter (CHDM) were also proposed in 1984, although not investigated in detail until the early 1990s.

The recent atmospheric neutrino data from Super-Kamiokande provide strong evidence of neutrino oscillations and therefore of non-zero neutrino mass. These data imply a lower limit on the HDM (i.e., light neutrino) contribution to the cosmological density $\Omega_\nu \gtrsim 0.001$ — almost as much as that of all the stars in the centers of galaxies — and permit higher Ω_ν . The “standard” COBE-normalized critical-matter-density (i.e., $\Omega_m = 1$) CDM model has too much power on small scales. It was discovered in 1992-95 that CDM with the addition of neutrinos with total mass of about 5 eV, corresponding to $\Omega_\nu \approx 0.2$, results in a much improved fit to data on the nearby galaxy and cluster distribution. Indeed, the resulting Cold + Hot Dark Matter (CHDM) cosmological model is arguably the most successful $\Omega_m = 1$ model for structure formation [1,2,3,4].

However, other recent data have begun to make a convincing case for $0.3 \lesssim \Omega_m \lesssim 0.5$. In light of all these new data, several authors have considered whether cosmology still provides evidence favoring neutrino mass of a few eV in flat models with cosmological constant $\Omega_\Lambda = 1 - \Omega_m$. The conclusion is that the possible improvement of the low- Ω_m flat (Λ CDM) cosmological models with the addition of light neutrinos appears to be rather limited, but that Λ CHDM models with $\Omega_\nu \lesssim 0.1$ may be consistent with presently available data. Data expected soon may permit detection of such a hot dark

matter contribution, or alternatively provide stronger upper limits on Ω_ν and neutrino masses.

2 Hot, Warm, and Cold Dark Matter

Hot DM refers to particles, such as neutrinos, that were moving at nearly the speed of light at redshift $z \sim 10^6$ (or time $t \sim 1$ yr), when the temperature $T \sim 3 \times 10^2$ eV and the cosmic horizon first encompassed $10^{12} M_\odot$, the amount of dark matter contained in the halo of a large galaxy like the Milky Way. Hot DM particles must also be still in thermal equilibrium after the last phase transition in the hot early universe, the QCD confinement transition, which presumably took place at $T_{\text{QCD}} \approx 10^2$ MeV. Hot DM particles have a cosmological number density roughly comparable to that of the microwave background photons, which (as we will see shortly) implies an upper bound to their mass of a few tens of eV. This then implies that free streaming of these relativistic particles destroys any fluctuations smaller than supercluster size, $\sim 10^{15} M_\odot$.

The “hot,” “warm,” “cold” DM terminology was introduced in 1983 [5,6]. Warm DM particles interact much more weakly than neutrinos. They decouple (i.e., their mean free path first exceeds the horizon size) at $T \gg T_{\text{QCD}}$, and are not heated by the subsequent annihilation of hadronic species. Consequently their number density is roughly an order of magnitude lower, and their mass an order of magnitude higher, than hot DM particles. Fluctuations corresponding to sufficiently large galaxy halos, $\gtrsim 10^{11} M_\odot$, could then survive free streaming. In theories of local supersymmetry broken at $\sim 10^6$ GeV, gravitinos could be DM of the warm variety [7,8,9]. Other warm dark matter candidates are also possible, of course, such as right-handed neutrinos [10]. Warm DM does not fit the observations if $\Omega_m = 1$ [11], but for low Ω_m some have suggested that it may be worth reconsidering, to avoid some possible problems of Cold DM [12,13]. However, the cutoff in the power spectrum $P(k)$ at large k implied by WDM will also inhibit the formation of small dark matter halos at high redshift. But such small halos are presumably where the first stars form, which produce metals rather uniformly throughout the early universe as indicated by observations of the Lyman α forest (neutral hydrogen clouds seen in absorption in quasar spectra).

Cold DM consists of particles for which free streaming is of no cosmological importance. Two different sorts of cold DM consisting of elementary particles have been proposed, heavy thermal remnants of annihilation such as supersymmetric neutralinos, and a cold Bose condensate such as axions. A universe where the matter is mostly cold DM and there is a large cosmological constant looks very much like the one astronomers actually observe, and this low- Ω_m Λ CDM model [14] is the current favorite model for structure formation in the universe [15,16,17].

3 Galaxy Formation with Hot DM

The standard hot DM candidate is massive neutrinos [18,19,20,21], although other, more exotic theoretical possibilities have been suggested, such as a “majoron” of nonzero mass which is lighter than the lightest neutrino species, and into which all neutrinos decay. Neutrinos appeared to be an attractive DM candidate because of the measurement of an electron neutrino mass of about 30 eV in 1980 [22]. This coincided with the improving CMB limits on the primordial fluctuation amplitude, which forced Zel’dovich and other theorists to abandon the idea that all the dark matter could be made of ordinary baryonic matter. The version of HDM that they worked out in detail, with adiabatic Gaussian primordial fluctuations, became the prototype for the subsequent $\Omega_m = 1$ CDM theory.

3.1 Mass Constraints

Direct measurements of neutrino masses have given only upper limits (see also the chapter by Robertson and Wilkerson). A secure upper limit on the electron neutrino mass is roughly 15 eV. The Particle Data Group [23] notes that a more precise limit cannot be given since unexplained effects have resulted in significantly negative measurements of $m(\nu_e)^2$ in tritium beta decay experiments. However, this problem is at least partially resolved, and the latest experimental upper limits on the electron neutrino mass are 2.8 eV from the Mainz [24] and 2.5 eV from the Troitsk [25] tritium beta decay experiments (both 95% C.L.). There is an upper limit on an effective Majorana neutrino mass of ~ 1 eV from neutrinoless double beta decay experiments [26]. The upper limits from accelerator experiments on the masses of the other neutrinos are $m(\nu_\mu) < 0.17$ MeV (90% CL) and $m(\nu_\tau) < 18$ MeV (95% CL) [23,27], but since stable neutrinos with such large masses would certainly “overclose the universe” (i.e., contribute such a large cosmological density that the universe could never have attained its present age), cosmology implies a much lower upper limit on these neutrino masses.

Before going further, it will be necessary to discuss the thermal history of neutrinos in the standard hot big bang cosmology in order to derive the corresponding constraints on their mass. Left-handed neutrinos of mass ≤ 1 MeV remain in thermal equilibrium until the temperature drops to $T_{\nu d}$, at which point their mean free path first exceeds the horizon size and they essentially cease interacting thereafter, except gravitationally [28]. Their mean free path is, in natural units ($\hbar = c = 1$), $\lambda_\nu \sim [\sigma_\nu n_{e^\pm}]^{-1} \sim [(G_F^2 T^2)(T^3)]^{-1}$, where $G_F \approx 10^{-5}$ GeV⁻² is the Fermi constant that measures the strength of the weak interactions. The horizon size is $\lambda_h \sim (G\rho)^{-1/2} \sim M_{Pl} T^{-2}$, where the Planck mass $M_{Pl} \equiv G^{-1/2} = 1.22 \times 10^{19}$ GeV. Thus $\lambda_h/\lambda_\nu \sim (T/T_{\nu d})^3$, with the neutrino decoupling temperature

$$T_{\nu d} \sim M_{Pl}^{-1/3} G_F^{-2/3} \sim 1 \text{ MeV} . \quad (1)$$

After T drops below $\frac{1}{2}$ MeV, e^+e^- annihilation ceases to be balanced by pair creation, and the entropy of the e^+e^- pairs heats the photons. Above 1 MeV, the number density $n_{\nu i}$ of each left-handed neutrino species and its right-handed antiparticle is equal to that of the photons, n_γ , times the factor $3/4$ from Fermi versus Bose statistics. But then e^+e^- annihilation increases the photon number density relative to that of the neutrinos by a factor of $11/4$.⁽¹⁾ As a result, the neutrino temperature $T_{\nu,0} = (4/11)^{1/3}T_{\gamma,0}$. Thus today, for each species,

$$n_{\nu,0} = \frac{3}{4} \cdot \frac{4}{11} n_{\gamma,0} = 109 \theta^3 \text{cm}^{-3}, \quad (2)$$

where $\theta \equiv (T_0/2.7\text{K})$. With the cosmic background radiation temperature $T_0 = 2.728 \pm 0.004$ K measured by the FIRAS instrument on the COBE satellite [29], $T_{\nu,0} = 1.947$ K and $n_{\nu,0} = 112 \text{cm}^{-3}$.

Since the present cosmological matter density is

$$\bar{\rho}_m = \Omega \rho_c = 10.54 \Omega_m h^2 \text{keV cm}^{-3}, \quad (3)$$

it follows that

$$\sum_i m_{\nu i} < \bar{\rho}_m / n_{\nu,0} \leq 96 \Omega_m h^2 \theta^{-3} \text{eV} \approx 93 \Omega_m h^2 \text{eV}, \quad (4)$$

where the sum runs over all neutrino species with $M_{\nu i} \leq 1$ MeV. (Heavier neutrinos will be discussed in the next paragraph.) Observational data imply that $\Omega_m h^2 \approx 0.1 - 0.3$, since $\Omega_m \approx 0.3 - 0.5$ and $h \approx 0.65 \pm 0.1$ [17]. Thus if all the dark matter were light neutrinos, the sum of their masses would be $\approx 9 - 28$ eV.

In deriving eq. (4), we have been assuming that all the neutrino species are light enough to still be relativistic at decoupling, i.e. lighter than an MeV. The bound (4) shows that they must then be much lighter than that. In the alternative case that a neutrino species is nonrelativistic at decoupling, it has been shown [30,31,32,33,34] that its mass must then exceed several GeV, which is not true of the known neutrinos (ν_e , ν_μ , and ν_τ). (One might at first think that the Boltzmann factor would sufficiently suppress the number

¹ In the argument giving the $11/4$ factor, the key ingredient is that the entropy in interacting particles in a comoving volume S_I is conserved during ordinary Hubble expansion, even during a process such as electron-positron annihilation, so long as it occurs in equilibrium. That is, $S_I = g_I(T)N_\gamma(T) = \text{constant}$, where $N_\gamma = n_\gamma V$ is the number of photons in a given comoving volume V , and $g_I = (g_B + \frac{7}{8}g_F)_I$ is the effective number of helicity states in interacting particles (with the factor of $\frac{7}{8}$ reflecting the difference in energy density for fermions versus bosons). Just above the temperature of electron-positron annihilation, $g_I = g_\gamma + \frac{7}{8} \times g_e = 2 + \frac{7}{8} \times 4 = \frac{11}{2}$; while below it, $g_I = g_\gamma = 2$. Thus, as a result of the entropy of the electrons and positrons being dumped into the photon gas at annihilation, the photon number density is thereafter increased relative to that of the neutrinos by a factor of $11/4$.

density of neutrinos weighing a few tens of MeV to allow compatibility with the present density of the universe. It is the fact that they “freeze out” of equilibrium well before the temperature drops to their mass that leads to the higher mass limit.) We have also been assuming that the neutrino chemical potential is negligible, i.e. that $|n_\nu - n_{\bar{\nu}}| \ll n_\gamma$. This is very plausible, since the net baryon number density ($n_b - n_{\bar{b}} \lesssim 10^{-9} n_\gamma$, and big bang nucleosynthesis restricts the allowed parameters [35] (see also the chapter by Fuller).

3.2 Phase Space Constraint

We have just seen that light neutrinos must satisfy an upper bound on the sum of their masses. But now we will discuss a lower bound on neutrino mass that arises because they must be rather massive to form the dark matter in galaxies, since their phase space density is limited by the Pauli exclusion principle. A slightly stronger bound follows from the fact that they were not degenerate in the early universe.

The phase space constraint [36] follows from Jeans’s theorem in classical mechanics to the effect that the maximum 6-dimensional phase space density cannot increase as a system of collisionless particles evolves. At early times, before density inhomogeneities become nonlinear, the neutrino phase space density is given by the Fermi-Dirac distribution

$$n_\nu(p) = \frac{g_\nu}{h^3} \left[1 + \exp\left(\frac{pc}{kT_\nu(z)}\right) \right]^{-1}, \quad (5)$$

where here h is Planck’s constant and $g_\nu = 2$ for each species of left-handed ν plus right-handed $\bar{\nu}$. Since momentum and temperature both scale as redshift z as the universe expands, this distribution remains valid after neutrinos drop out of thermal equilibrium at ~ 1 MeV, and even into the nonrelativistic regime $T_\nu < m_\nu$ [28]. The standard version of the phase space constraint follows from demanding that the central phase space density $9[2(2\pi)^{5/2}Gr_c^2\sigma m_\nu^4]^{-1}$ of the DM halo, assumed to be an isothermal sphere of core radius r_c and one-dimensional velocity dispersion σ , not exceed the maximum value of the initial phase space density $n_\nu(0) = g_\nu/2h^3$. The result is

$$m_\nu > (120 \text{ eV}) \left(\frac{100 \text{ km s}^{-1}}{\sigma} \right)^{1/4} \left(\frac{1 \text{ kpc}}{r_c} \right)^{1/2} \left(\frac{g_\nu}{2} \right)^{-1/4}. \quad (6)$$

The strongest lower limits on m_ν follow from applying this to the smallest galaxies. Both theoretical arguments regarding the dwarf spheroidal (dS) satellite galaxies of the Milky Way [37] and data on Draco, Carina, and Ursa Minor made it clear some time ago that dark matter dominates the gravitational potential of these dS galaxies, and the case has only strengthened with time [38]. The phase space constraint then sets a lower limit [39] $m_\nu >$

500 eV, which is completely incompatible with the cosmological constraint eq. (4). However, this argument only excludes neutrinos as the DM in certain small galaxies; it remains possible that the DM in these galaxies is (say) baryonic, while that in larger galaxies such as our own is (at least partly) light neutrinos. A more conservative phase space constraint was obtained for the Draco and Ursa Minor dwarf spheroidals [40], but the authors concluded that neutrinos consistent with the cosmological upper bound on m_ν cannot be the DM in these galaxies. A similar analysis applied to the gas-rich low-rotation-velocity dwarf irregular galaxy DDO 154 [41] gave a limit $m_\nu > 94$ eV, again inconsistent with the cosmological upper bound.

3.3 Free Streaming

The most salient feature of hot DM is the erasure of small fluctuations by free streaming. Thus even collisionless particles effectively exhibit a Jeans mass. It is easy to see that the minimum mass of a surviving fluctuation is of order $M_{P\ell}^3/m_\nu^2$ [42,43]. Let us suppose that some process in the very early universe — for example, thermal fluctuations subsequently vastly inflated in the inflationary scenario — gave rise to adiabatic fluctuations on all scales. In adiabatic fluctuations, all the components — radiation and matter — fluctuate together. Neutrinos of nonzero mass m_ν stream relativistically from decoupling until the temperature drops to $T \sim m_\nu$, during which time they traverse a distance $d_\nu = R_H(T = m_\nu) \sim M_{P\ell} m_\nu^{-2}$. In order to survive this free streaming, a neutrino fluctuation must be larger in linear dimension than d_ν . Correspondingly, the minimum mass in neutrinos of a surviving fluctuation is $M_{J,\nu} \sim d_\nu^3 m_\nu n_\nu(T = m_\nu) \sim d_\nu^3 m_\nu^4 \sim M_{P\ell}^3 m_\nu^{-2}$. By analogy with Jeans's calculation of the minimum mass of an ordinary fluid perturbation for which gravity can overcome pressure, this is referred to as the (free-streaming) Jeans mass.

A more careful calculation [43,44] gives

$$d_\nu = 41(m_\nu/30 \text{ eV})^{-1}(1+z)^{-1} \text{Mpc} , \quad (7)$$

that is, $d_\nu = 41(m_\nu/30\text{eV})^{-1}$ Mpc in comoving coordinates, and correspondingly

$$M_{J,\nu} = 1.77 M_{P\ell}^3 m_\nu^{-2} = 3.2 \times 10^{15} (m_\nu/30 \text{ eV})^{-2} M_\odot , \quad (8)$$

which is the mass scale of superclusters. Objects of this size are the first to form in a ν -dominated universe, and smaller scale structures such as galaxies can form only after the initial collapse of supercluster-size fluctuations.

When a fluctuation of total mass $\sim 10^{15} M_\odot$ enters the horizon at $z \sim 10^4$, the density contrast δ_{RB} of the radiation plus baryons ceases growing and instead starts oscillating as an acoustic wave, while that of the massive neutrinos δ_ν continues to grow linearly with the scale factor $R = (1+z)^{-1}$ since the Compton drag that prevents growth of δ_{RB} does not affect the neutrinos.

By recombination, at $z_r \sim 10^3$, $\delta_{\text{RB}}/\delta_\nu \lesssim 10^{-1}$, with possible additional suppression of δ_{RB} by Silk damping. Thus the hot DM scheme with adiabatic primordial fluctuations predicts small-angle fluctuations in the microwave background radiation that are lower than in the adiabatic baryonic cosmology, which was one of the reasons HDM appealed to Zel'dovich and other theorists. Similar considerations apply in the warm and cold DM schemes. However, as we will discuss in a moment, the HDM top-down sequence of cosmogony is wrong, and with the COBE normalization hardly any structure would form by the present.

In numerical simulations of dissipationless gravitational clustering starting with a fluctuation spectrum appropriately peaked at $\lambda \sim d_\nu$ (reflecting damping by free streaming below that size and less time for growth of the fluctuation amplitude above it), the regions of high density form a network of filaments, with the highest densities occurring at the intersections and with voids in between [45,46,47,48]. The similarity of these features to those seen in observations was cited as evidence in favor of HDM [49].

3.4 Problems with ν DM

A number of potential problems with the neutrino dominated universe had emerged by about 1983, however.

- From studies both of nonlinear clustering [48,50] (comoving length scale $\lambda \lesssim 10$ Mpc) and of streaming velocities [51] in the linear regime ($\lambda > 10$ Mpc), it follows that supercluster collapse must have occurred recently: $z_{sc} \leq 0.5$ is indicated and in any case $z_{sc} < 2$ [48]. However, the best limits on galaxy ages coming from globular clusters and other stellar populations indicated that galaxy formation took place before $z \approx 3$. Moreover, if quasars are associated with galaxies, as is suggested by the detection of galactic luminosity around nearby quasars and the apparent association of more distant quasars with galaxy clusters, the abundance of quasars at $z > 2$ was also inconsistent with the “top-down” neutrino dominated scheme in which superclusters form first: $z_{sc} > z_{galaxies}$.
- Numerical simulations of the nonlinear “pancake” collapse taking into account dissipation of the baryonic matter showed that at least 85% of the baryons are so heated by the associated shock that they remain unable to condense, attract neutrino halos, and eventually form galaxies [5,52]. This was a problem for the hot DM scheme for two reasons. With the primordial nucleosynthesis constraint $\Omega_b \lesssim 0.1$, there would be difficulty having enough baryonic matter condense to form the luminosity that we actually observe. And, where are the X-rays from the shock-heated pancakes [53]?
- The neutrino picture predicts [54] that there should be a factor of ~ 5 increase in M/M_b between large galaxies ($M \sim 10^{12}M_\odot$) and large clusters ($M \geq 10^{14}M_\odot$), since the larger clusters, with their higher escape

velocities, are able to trap a considerably larger fraction of the neutrinos. Although there is some indication that the mass-to-light ratio M/L increases with M , the ratio of total to luminous mass M/M_{lum} is probably a better indicator of the value of M/M_b , and it is roughly the same for galaxies with large halos and for rich clusters.

These problems, while serious, would perhaps not have been fatal for the hot DM scheme. But an even more serious problem for HDM arose from the low amplitude of the CMB fluctuations detected by the COBE satellite, $(\Delta T/T)_{rms} = (1.1 \pm 0.2) \times 10^{-5}$ smoothed on an angular scale of about 10° [55]. Although HDM and CDM both have the Zel'dovich spectrum shape ($P(k) \propto k$) in the long-wavelength limit, because of the free-streaming cutoff the amplitude of the HDM spectrum must be considerably higher in order to form any structure by the present. With the COBE normalization, the HDM spectrum is only beginning to reach nonlinearity at the present epoch.

Thus the evidence against standard hot DM is convincing. At very least, it indicates that structure formation in a neutrino-dominated universe must be rather more complicated than in the standard inflationary picture.

The main alternative that has been considered is cosmic strings plus hot dark matter. Because the strings would continue to seed structure up until the present, and because these seeds are in the nature of rather localized fluctuations, hot DM would probably work better with string seeds than cold DM. However, strings and other cosmic defect models are now essentially ruled out [56,57] because they predict that the cosmic microwave background would have an angular power spectrum without the pronounced (doppler/acoustic/Sakharov) peak at angular wavenumber $l \sim 220$ that now appears to be clearly indicated by the data, along with secondary peaks at higher l .

4 Cold plus Hot Dark Matter and Structure Formation: $\Omega_m = 1$

Even if most of the dark matter is of the cold variety, a little hot dark matter can have a dramatic effect on the predicted distribution of galaxies. In the early universe, the free streaming of the fast-moving neutrinos washes out any inhomogeneities in their spatial distribution on the scales that will later become galaxies. If these neutrinos are a significant fraction of the total mass of the universe, then although the density inhomogeneities will be preserved in the cold dark matter, their growth rates will be slowed. As a result, the amplitude of the galaxy-scale inhomogeneities today is less with a little hot dark matter than if the dark matter is only cold. (With the tilt n of the primordial spectrum $P_p(k) = Ak^n$ fixed — which as we discuss below is not necessarily reasonable — the fractional reduction in the power on small scales is $\Delta P/P \approx 8\Omega_\nu/\Omega_m$ [58]. See Fig. 1 for examples of how the power spectrum

$P(k)$ is affected by the addition of hot dark matter in $\Omega_m = 0.4$ flat cosmologies.) Since the main problem with $\Omega_m = 1$ cosmologies containing only cold dark matter is that the amplitude of the galaxy-scale inhomogeneities is too large compared to those on larger scales, the presence of a little hot dark matter appeared to be possibly just what was needed. And, as was mentioned at the outset, a CHDM model with $\Omega_m = 1$, $\Omega_\nu = 0.2$, and Hubble parameter $h = 0.5$ is perhaps the best fit to the galaxy distribution in the nearby universe of any cosmological model. The effects of the relatively small amount of hot dark matter in a CHDM model on the distribution of matter compared to a purely CDM model are shown graphically in [59]; cf. also [60]. As expected, within galaxy halos the distribution of cold and hot particles is similar. But the hot particles are more widely distributed on larger scales, and the hot/cold ratio is significantly enhanced in low-density regions.

The first step in working out the theory of structure formation is to use linear perturbation theory, which is valid since cosmic microwave background measurements show that density fluctuations are small at the redshift of recombination, $z_r \sim 10^3$. The most extensive early calculations of this sort were carried out by Holtzman [61,62], who concluded that the most promising cosmological models were CHDM and Λ CDM [63]. The most efficient method of computing the linear evolution of fluctuations now is that used in the CMBFAST code [64]. An alternative Monte Carlo treatment of the evolution of neutrino density fluctuations was given by [65], but the differences from the usual treatment appear to be small. Detailed analytic results have been given by [66,67] and reviewed in [60]. But the key point can be understood simply: there is less structure in CHDM models on small scales because the growth rate of cold dark matter fluctuations is reduced on the scales where free streaming has wiped out neutrino fluctuations. Let us define the fluctuation growth rate f by

$$f(k) \equiv \frac{d \log \delta(k)}{d \log a}, \quad (9)$$

where $\delta(k)$ is the amplitude of the fluctuations of wave number $k = 2\pi/\lambda$ in cold dark matter, and as usual $a = 1/(1+z)$ is the scale factor. For $\Omega_m = 1$ CDM fluctuations, the growth rate $f = 1$. This is also true for fluctuations in CHDM, for k sufficiently small that free-streaming has not significantly decreased the amplitude of neutrino fluctuations. However, in the opposite limit $k \rightarrow \infty$ [43,60],

$$f_\infty = (\sqrt{1 + 24\Omega_c} - 1)/4 \approx \Omega_c^{0.6}, \quad (10)$$

assuming that $\Omega_c + \Omega_\nu = 1$. For example, for $\Omega_\nu = 0.2$, $f_\infty = 0.87$. Even though the growth rate is only a little lower for these large- k (i.e., short-wavelength) modes, the result is that their amplitude is decreased substantially compared to longer-wavelength modes.

The next step in determining the implications for structure formation is to work out the effects on nonlinear scales using N-body simulations. This

is harder for Cold+Hot models than for CDM because the higher velocities of the neutrinos require more particles to adequately sample the neutrino phase space. The simulations must reflect the fact that the neutrinos initially have a redshifted Fermi-Dirac phase space distribution [68]. These CHDM simulations were compared with observational data using various statistics. CHDM with $\Omega_\nu = 0.3$, the value indicated by approximate analyses [63,69], was shown to lead to groups of galaxies having substantially lower velocity dispersions than CDM, and in better agreement with observations [70]. But it also leads to a Void Probability Function (VPF) with more intermediate-sized voids than are observed [71]. This theory had so little small-scale power that a quasi-linear analysis using the Press-Schechter approximation showed that there would not be enough of the high-column-density hydrogen clouds at high redshift $z \sim 3$ known as damped Lyman- α systems [72,73,74]. But CHDM with $\Omega_\nu = 0.2$ suppresses small-scale fluctuations less and therefore has a better chance of avoiding this problem [75]. Simulations [76] showed that this version of CHDM also has a VPF in good agreement with observations [77]. The group velocity dispersions also remained sufficiently small to plausibly agree with observations, but it had become clear that the N-body simulations used lacked sufficient resolution to identify galaxies so that this statistic could be measured reliably [78].

A resolution problem also arose regarding the high-redshift damped Lyman- α systems. Earlier research had been based on the idea that these systems are rather large disk galaxies in massive halos [79], but then high-resolution hydrodynamical simulations [80] showed that relatively small gaseous protogalaxies moving in smaller halos provide a good match to the new, detailed kinematic data [81]. It thus appeared possible that CHDM models with $\Omega_\nu \lesssim 0.2$ might produce enough damped Lyman- α systems. With the low Hubble parameter $h \sim 0.5$ required for such $\Omega_m = 1$ models, the total neutrino mass would then be $\lesssim 5$ eV.

While neutrino oscillation experiments can determine the differences of squared neutrino masses, as we will briefly review next, cosmology is sensitive to the actual values of the neutrino masses — for any that are larger than about 1 eV. In that case, cosmology can help to fill in the neutrino mass matrix.

One example of this is the fact that if the hot DM mass is roughly evenly shared between two or three neutrino species, the neutrinos will be lighter than if the same mass were all in one species, so that the free streaming length will be longer. A consequence is that, for the same total neutrino mass and corresponding Ω_ν , the power spectrum will be approximately 20% lower on the scale of galaxy clusters if the mass is shared between two neutrino species [1]. Since the amplitude and “tilt” n of the power spectrum in CDM-type models is usually fixed by comparison with COBE and cluster abundance, this has the further consequence that higher n (i.e., less tilt) is required when the neutrino mass is divided between comparable-mass neutrino species. Less

tilt means that there will be more power on small scales, which appeared to be favorable for the CHDM model, for example because it eased the problems with damped Lyman- α systems [1,82].

5 Evidence for Neutrino Mass from Oscillations

There is mounting astrophysical and laboratory data suggesting that neutrinos oscillate from one species to another [27], which can only happen if they have non-zero mass. Of these experimental results, the ones that are regarded as probably most secure are those concerning atmospheric neutrino oscillations from Super-Kamiokande (see the chapter by John Learned) and solar neutrinos from several experiments (see the chapter by Wick Haxton). But the experimental results that are most relevant to neutrinos as hot dark matter are from the Liquid Scintillator Neutrino Detector (LSND) experiment at Los Alamos (see the chapter by David Caldwell).

Older Kamiokande data [83] showed that, for events attributable to atmospheric neutrinos with visible energy $E > 1.3$ GeV, the deficit of ν_μ increases with zenith angle. The Super-Kamiokande detector has confirmed and extended the results of its smaller predecessor [84]. These data imply that $\nu_\mu \rightarrow \nu_\tau$ oscillations occur with a large mixing angle $\sin^2 2\theta > 0.82$ and an oscillation length several times the height of the atmosphere, which implies that $5 \times 10^{-4} < \Delta m_{\tau\mu}^2 < 6 \times 10^{-3}$ eV² (90% CL). (Neutrino oscillation experiments measure not the masses, but rather the difference of the squared masses, of the oscillating species, here $\Delta m_{\tau\mu}^2 \equiv |m(\nu_\tau)^2 - m(\nu_\mu)^2|$.) This in turn implies that if other data requires either ν_μ or ν_τ to have large enough mass ($\gtrsim 1$ eV) to be a hot dark matter particle, then they must be nearly equal in mass, i.e., the hot dark matter mass would be shared between these two neutrino species. Both the new Super-Kamiokande atmospheric ν_e data and the lack of a deficit of $\bar{\nu}_e$ in the CHOOZ reactor experiment [85] make it quite unlikely that the atmospheric neutrino oscillation is $\nu_\mu \rightarrow \nu_e$. If the oscillation were instead to a sterile neutrino, the large mixing angle implies that this sterile species would become populated in the early universe and lead to too much ⁴He production during the Big Bang Nucleosynthesis epoch [86]. (Sterile neutrinos are discussed further below.) It may be possible to verify that $\nu_\mu \rightarrow \nu_\tau$ oscillations occur via a long-baseline neutrino oscillation experiment. The K2K experiment is looking for missing ν_μ due to $\nu_\mu \rightarrow \nu_\tau$ oscillations with a beam of ν_μ from the Japanese KEK accelerator directed at the Super-Kamiokande detector, with more powerful Fermilab-Soudan and CERN-Gran Sasso long-baseline experiments in preparation, the latter of which will look for τ appearance.

The observation by LSND of events that appear to represent $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ oscillations followed by $\bar{\nu}_e + p \rightarrow n + e^+$, $n + p \rightarrow D + \gamma$, with coincident detection of e^+ and the 2.2 MeV neutron-capture γ -ray, suggests that $\Delta m_{\mu e}^2 > 0$ [87]. The independent LSND data [88] suggesting that $\nu_\mu \rightarrow \nu_e$ oscillations

are also occurring is consistent with, but has less statistical weight than, the LSND signal for $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ oscillations. Comparison of the latter with exclusion plots from other experiments allows two discrete values of $\Delta m_{\mu e}^2$, around 10.5 and 5.5 eV², or a range $2 \text{ eV}^2 \gtrsim \Delta m_{\mu e}^2 \gtrsim 0.2 \text{ eV}^2$. The lower limit in turn implies a lower limit $m_\nu \gtrsim 0.5 \text{ eV}$, or $\Omega_\nu \gtrsim 0.01(0.65/h)^2$. This would imply that the contribution of hot dark matter to the cosmological density is at least as great as that of all the visible stars $\Omega_* \approx 0.0045(0.65/h)$ [89]. Such an important conclusion requires independent confirmation. The KARlsruhe Rutherford Medium Energy Neutrino (KARMEN) experiment has added shielding to decrease its background so that it can probe a similar region of $\Delta m_{\mu e}^2$ and neutrino mixing angle; the KARMEN results exclude a significant portion of the LSND parameter space, and the numbers quoted above take into account the current KARMEN limits. The Booster Neutrino Experiment (BOONE) at Fermilab should attain greater sensitivity.

The observed deficit of solar electron neutrinos in three different types of experiments suggests that some of the ν_e undergo Mikheyev-Smirnov-Wolfenstein matter-enhanced oscillations $\nu_e \rightarrow \nu_x$ to another species of neutrino ν_x with $\Delta m_{ex}^2 \approx 10^{-5} \text{ eV}^2$ as they travel through the sun [90], or possibly “Just-So” vacuum oscillations with even smaller Δm_{ex}^2 [91]. The LSND $\nu_\mu \rightarrow \nu_e$ signal with a much larger $\Delta m_{e\mu}^2$ is inconsistent with $x = \mu$, and the Super-Kamiokande atmospheric neutrino oscillation data is inconsistent with $x = \tau$. Thus a fourth neutrino species ν_s is required if all these neutrino oscillations are actually occurring. Since the neutral weak boson Z^0 decays only to three species of neutrinos, any additional neutrino species ν_s could not couple to the Z^0 , and is called “sterile.” This is perhaps distasteful, although many modern theories of particle physics beyond the standard model include the possibility of such sterile neutrinos. The resulting pattern of neutrino masses would have ν_e and ν_s very light, and $m(\nu_\mu) \approx m(\nu_\tau) \approx (\Delta m_{e\mu}^2)^{1/2}$, with the ν_μ and ν_τ playing the role of the hot dark matter particles if their masses are high enough [92]. This neutrino spectrum might also explain how heavy elements are synthesized in core-collapse supernova explosions [93]. Note that the required solar neutrino mixing angle is very small, unlike that required to explain the atmospheric ν_μ deficit, so a sterile neutrino species would not be populated in the early universe and would not lead to too much ⁴He production.

Of course, if one or more of the indications of neutrino oscillations are wrong, then a sterile neutrino would not be needed and other patterns of neutrino masses are possible. But in any case the possibility remains of neutrinos having large enough mass to be hot dark matter. Assuming that the Super-Kamiokande data on atmospheric neutrinos are really telling us that ν_μ oscillates to ν_τ , the two simplest possibilities regarding neutrino masses are as follows:

A) Neutrino masses are hierarchical like all the other fermion masses, increasing with generation, as in see-saw models. Then the Super-Kamiokande $\Delta m^2 \approx 0.003$ implies $m(\nu_\tau) \approx 0.05$ eV, corresponding to

$$\Omega_\nu = 0.0013(m_\nu/0.05\text{eV})(0.65/h)^2. \quad (11)$$

This is not big enough to affect galaxy formation significantly, but it is another puzzling cosmic coincidence that it is close to the contribution to the cosmic density from stars.

B) The strong mixing between the mu and tau neutrinos implied by the Super-Kamiokande data suggests that these neutrinos are also nearly equal in mass, as in the Zee model [94] and many modern models [91,92] (although such strong mixing can also be explained in the context of hierarchical models based on the SO(10) Grand Unified Theory [95]). Then the above Ω_ν is just a lower limit. An upper limit is given by cosmological structure formation. In Cold + Hot Dark Matter (CHDM) models with $\Omega_m = 1$, we saw in the previous section that if Ω_ν is greater than about 0.2 the voids are too big and there is not enough early structure. In the next section we consider the upper limit on Ω_ν if $\Omega_m \approx 0.4$, which is favored by a great deal of data.

6 Cold plus Hot Dark Matter and Structure Formation: $\Omega_m \approx 0.4$

We have already mentioned that the $\Omega_m = 1$ CHDM model with $\Omega_\nu = 0.2$ was found to be the best fit to nearby galaxy data of all cosmological models [3]. But this didn't take into account the new high- z supernova data and analyses [96] leading to the conclusion that $\Omega_\Lambda - \Omega_{matter} \approx 0.2$, nor the new high-redshift galaxy data. Concerning the latter, Somerville, Primack, and Faber [97] found that none of the $\Omega_m = 1$ models with a realistic power spectrum (e.g., CHDM, tilted CDM, or τ CDM) makes anywhere near enough bright $z \sim 3$ galaxies. But we found that Λ CDM with $\Omega_m \approx 0.4$ makes about as many high-redshift galaxies as are observed [97]. This Ω_m value is also implied if clusters have the same baryon fraction as the universe as a whole: $\Omega_m \approx \Omega_b/f_b \approx 0.4$, using for the cosmological density of ordinary matter $\Omega_b = 0.019h^{-2}$ [98] and for the cluster baryon fraction $f_b = 0.06h^{-3/2}$ [99] from X-ray data or $f_b = 0.077h^{-1}$ from Sunyaev-Zel'dovich data [100]. An analysis of the cluster abundance as a function of redshift based on X-ray temperature data also implies that $\Omega_m \approx 0.44 \pm 0.12$ [101,102]. Thus most probably Ω_m is ~ 0.4 and there is a cosmological constant $\Omega_\Lambda \sim 0.6$. In the 1984 paper that helped launch CDM [14], we actually considered two models in parallel, CDM with $\Omega_m = 1$ and Λ CDM with $\Omega_m = 0.2$ and $\Omega_\Lambda = 0.8$, which we thought would bracket the possibilities. It looks like an Λ CDM intermediate between these extremes may turn out to be the right mix.

The success of $\Omega_m = 1$ CHDM in fitting the CMB and galaxy distribution data suggests that flat low- Ω_m cosmologies with a little hot dark matter

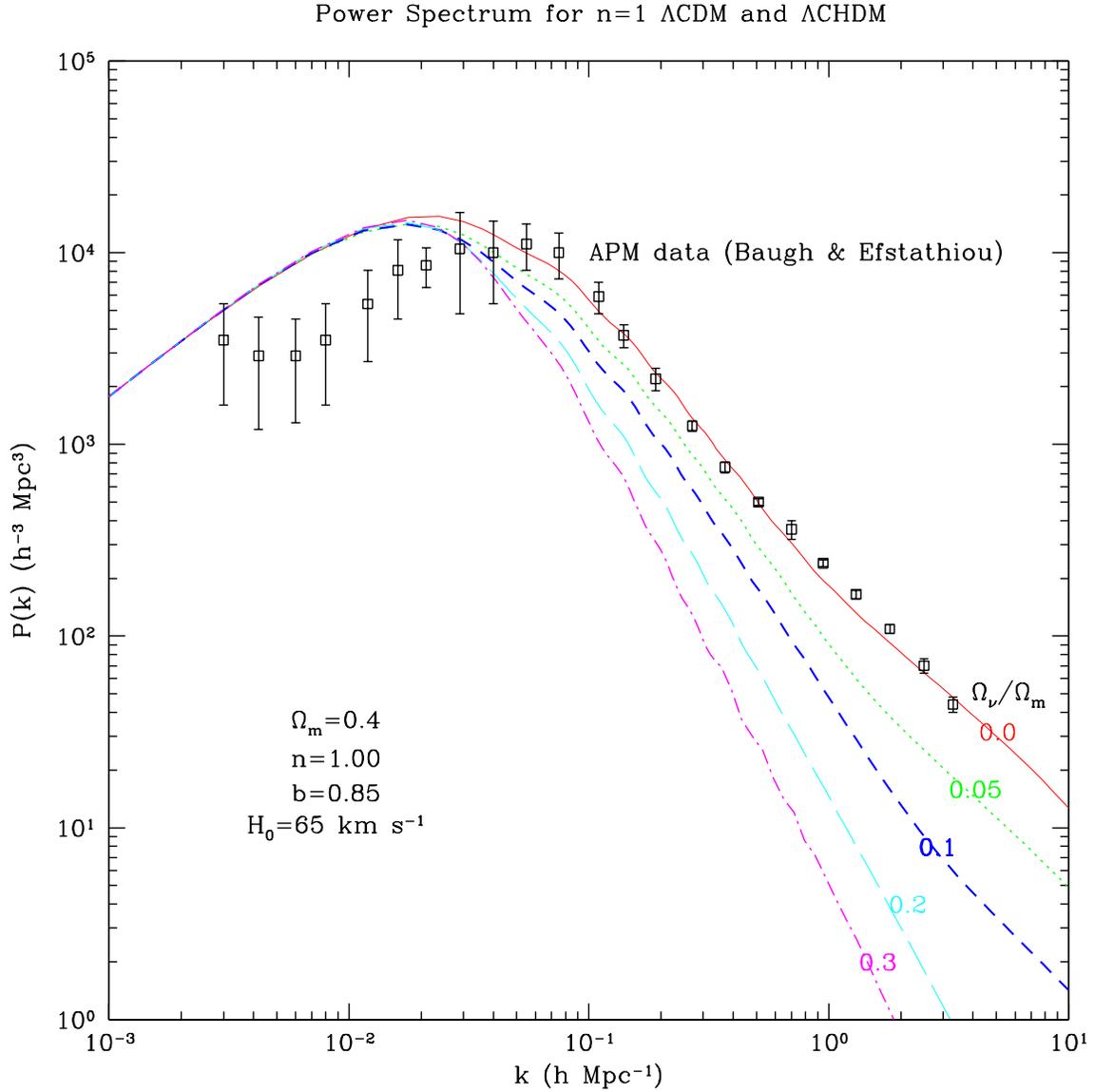


Fig. 1. Nonlinear dark matter power spectrum vs. wavenumber for Λ CDM and Λ CHDM models with $\Omega_\nu/\Omega_m = 0.05, 0.1, 0.2, 0.3$. Here $\Omega_m = 0.4$, the Hubble parameter $h = 0.65$, there is no tilt (i.e., $n = 1$), and the bias $b = 0.85$. Note that in this and the next Figure we “nonlinearized” all the model power spectra [103], to allow them all to be compared to the APM data (the small “wiggles” in the high- Ω_ν power spectra are an artifact of the nonlinearization procedure).

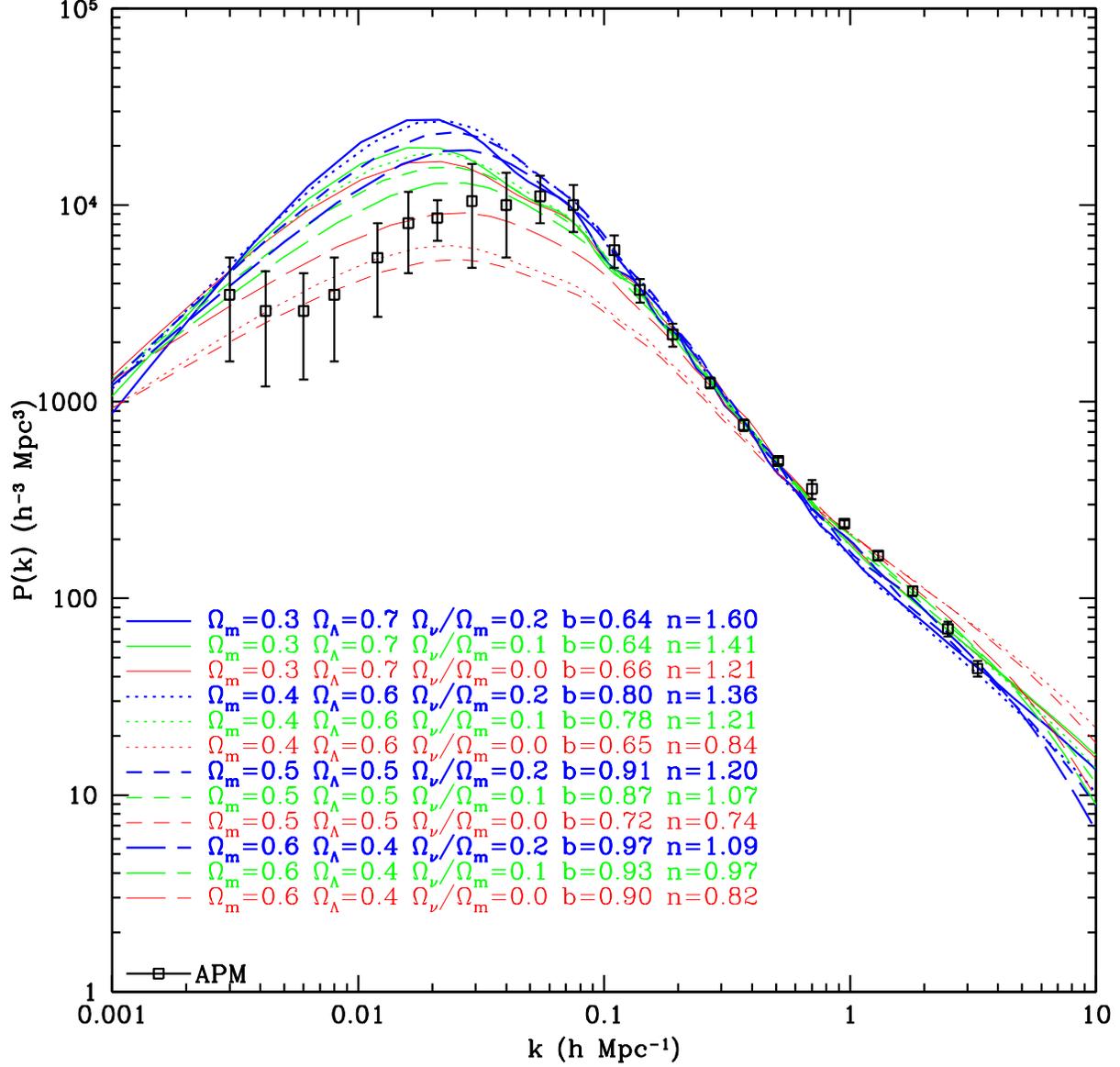


Fig. 2. Nonlinear dark matter power spectrum vs. wavenumber for 12 ACHDM models with $N_\nu = 2$ massive neutrino species and Hubble parameter $h = 0.65$, with tilt and σ_8 determined by COBE + ENACS cluster abundance. The bias chosen for these models is that which minimizes χ^2 over the entire range of available APM data.

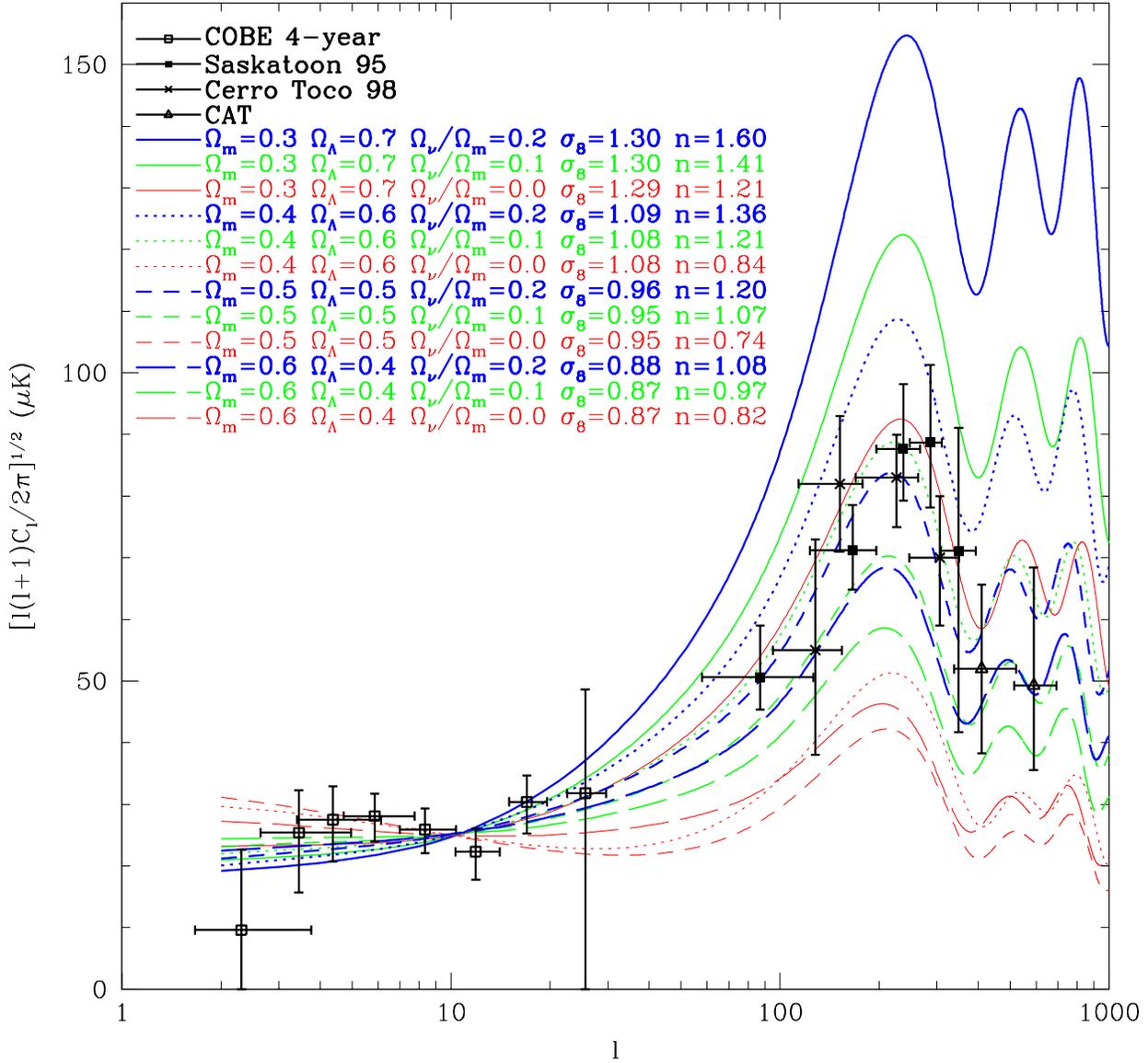


Fig. 3. CMB anisotropy power spectrum vs. angular wave number for the same models as in Figure 2. The data plotted are from COBE and three recent small-angle experiments [104,105,106,107].

be investigated in more detail. We have used CMBFAST [64] to examine Λ CHDM models with various h , Ω_m , and Ω_ν , assuming $\Omega_b = 0.019h^{-2}$. Figure 1 shows the power spectrum $P(k)$ for Λ CDM and a sequence of Λ CHDM models with increasing amounts of hot dark matter, compared to the power spectrum from APM [108]. Here we have fixed $\Omega_m = 0.4$ and Hubble parameter $h = 0.65$. All of these models have no tilt and the same bias parameter, to make it easier to compare them with each other. As expected, the large-scale power spectrum is the same for all these models, but the amount of small-scale power decreases as the amount of hot dark matter increases.

In Figures 2 and 3 we consider a sequence of twelve Λ CDM and Λ CHDM models with $h = 0.65$, $\Omega_m = 0.3, 0.4, 0.5, 0.6$, and $\Omega_\nu/\Omega_m = 0, 0.1$, and 0.2 . We have adjusted the amplitude and tilt n of the primordial power spectrum for each model in order to match the 4-year COBE amplitude and the ENACS differential mass function of clusters [109] (cf. [110]). (We checked the CMBFAST calculation of Λ CHDM models against Holtzman’s code used in our earlier investigation of Λ CHDM models [1]. Our results are also compatible with those of recent studies [111,112] in which $n = 1$ models were considered. But we find that some Λ CDM and Λ CHDM models require $n > 1$, called “anti-tilt”, and it is easy to create cosmic inflation models that give $n > 1$ — cf. [113].) In all the Λ CHDM models the neutrino mass is shared between $N_\nu = 2$ equal-mass species — as explained above, this is required by the atmospheric neutrino oscillation data if neutrinos are massive enough to be cosmologically significant hot dark matter. (This results in slightly more small-scale power compared to $N_\nu = 1$ massive species, as explained above, but the $N_\nu = 1$ curves are very similar to those shown.) In Ref. [114] we have shown similar results for Hubble parameter $h = 0.6$, and also plotted the best CHDM and Λ CDM models from [3]. Note that all these Figures are easier to read in color; see the version of this paper on the Los Alamos archive [115].

Of the Λ CHDM models shown, for $\Omega_m = 0.4 - 0.6$ the best simultaneous fits to the small-angle CMB and the APM galaxy power spectrum data [108] are obtained for the model with $\Omega_\nu/\Omega_m = 0.1$, and correspondingly $m(\nu_\mu) \approx m(\nu_\tau) \approx 0.8 - 1.2$ eV for $h = 0.65$. For $\Omega_m < 0.4$, smaller or vanishing neutrino mass appears to be favored. Note that the anti-tilt permits some of the Λ CHDM models to give a reasonably good fit to the COBE plus small-angle CMB data. Thus, adding a little hot dark matter to the moderate- Ω_m Λ CDM models may perhaps improve somewhat their simultaneous fit to the CMB and galaxy data, but the improvement is not nearly as dramatic as was the case for $\Omega_m = 1$.

It is apparent that the Λ CDM models with $\Omega_m = 0.4, 0.5$ have too much power at small scales ($k \gtrsim 1h^{-1}$ Mpc), as is well known [117,118] — although recent work [119] suggests that the distribution of *dark matter halos* in the $\Omega_m = 0.3$, $h = 0.7$ Λ CDM model may agree well with the APM data. On the other hand, the Λ CHDM models may have too little power on small scales — high-resolution Λ CHDM simulations and semi-analytic models of early

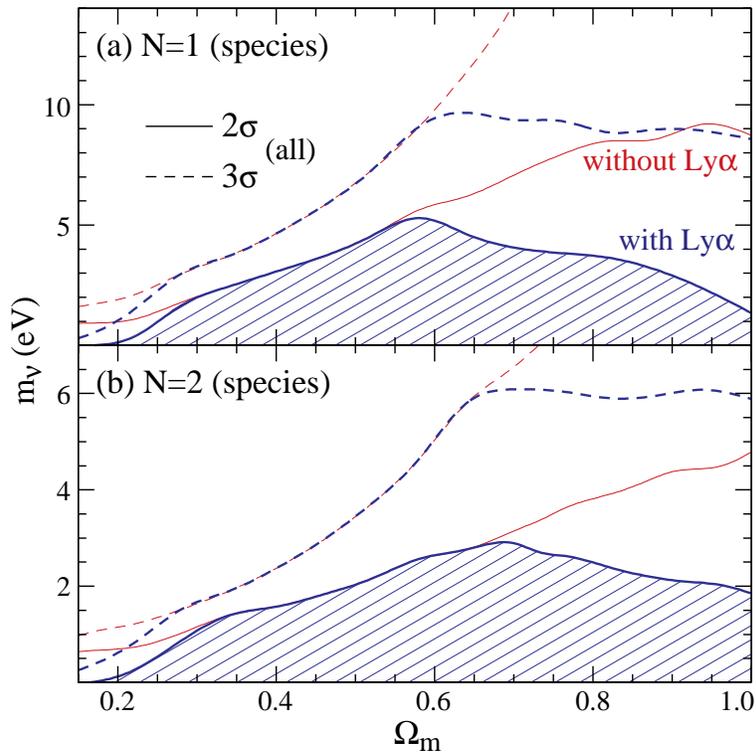


Fig. 4. Constraints on the neutrino mass assuming (a) $N_\nu = 1$ massive neutrino species and (b) $N_\nu = 2$ equal-mass neutrino species. The heavier weight curves show the effect of including the Lyman- α forest constraint. (From [116]; used by permission.)

galaxy formation may be able to clarify this. Such simulations should also be compared to data from the massive new galaxy redshift surveys 2dF and SDSS using shape statistics, which have been shown to be able to discriminate between CDM and CHDM models [120].

Note that all the Λ CDM and Λ CHDM models that are normalized to COBE and have tilt compatible with the cluster abundance are a poor fit to the APM power spectrum near the peak. The Λ CHDM models all have the peak in their linear power spectrum $P(k)$ higher and at lower k than the currently available data (e.g., from APM). Thus the viability of Λ CDM or Λ CHDM models with a power-law primordial fluctuation spectrum (i.e., just tilt n) depends on this data/analysis being wrong. In fact, it has recently been argued [121] that because of correlations, the errorbars underestimate the true errors in $P(k)$ for small k by at least a factor of 2. The new large-scale surveys 2dF and SDSS will be crucial in giving the first really reliable data on this, perhaps as early as next year.

The best published constraint on Ω_ν in Λ CDM models is [116]. Figure 4 shows the result of their analysis, which uses the COBE and cluster data much as we did above, the $P(k)$ data only for $0.025(h/\text{Mpc}) < k < 0.25(h/\text{Mpc})$, the constraint that the age of the universe is at least 13.2 ± 2.9 Gyr (95% C.L.) from globular clusters [122], and also the power spectrum at high redshift $z \sim 2.5$ determined from Lyman- α forest data. The conclusion is that the total neutrino mass m_ν is less than about 5.5 eV for all values of Ω_m , and $m_\nu \lesssim 2.4(\Omega_m/0.17 - 1)$ eV for the observationally favored range $0.2 \leq \Omega_m \leq 0.5$ (both at 95% C.L.). Analysis of additional Lyman- α forest data can allow detection of the signature of massive neutrinos even if m_ν is only a fraction of an eV. Useful constraints on Ω_ν will also come from large-scale weak gravitational lensing data [123] combined with cosmic microwave background anisotropy data.

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References

1. J.R. Primack, J. Holtzman, A.A. Klypin, D.O. Caldwell: Phys. Rev. Lett. 74, 2160 (1995)
2. J.R. Primack: ‘The Best Theory of Cosmic Structure Formation is Cold + Hot Dark Matter’. In *Critical Dialogues in Cosmology*, ed. by N. Turok (World Scientific, Singapore 1997), p. 535; updated in astro-ph/9707285
3. E. Gawiser, J. Silk: Science 280, 1405 (1998); cf. J.R. Primack: Science 280, 1398 (1998)
4. M.A.K. Gross, R. Somerville, J.R. Primack, J. Holtzman, A.A. Klypin: MNRAS, 301, 81 (1998)
5. J.R. Bond, J. Centrella, A.S. Szalay, J.R. Wilson: ‘Dark matter and shocked pancakes’. In *Formation and Evolution of Galaxies and Large Structures in the Universe*, ed. by J. Audouze, J. Tran Thanh Van (Reidel, Dordrecht 1984) pp. 87-99.
6. J.R. Primack, G.R. Blumenthal: ‘What is the Dark Matter?’. In *Formation and Evolution of Galaxies and Large Structures in the Universe*, ed. by J. Audouze, J. Tran Thanh Van (Reidel, Dordrecht 1984) pp. 163-83.
7. H. Pagels, J.R. Primack: Phys. Rev. Lett. 48, 223 (1982)
8. G.R. Blumenthal, H. Pagels, J.R. Primack: Nature 299, 37 (1982)
9. J.R. Bond, A.S. Szalay, M.S. Turner: Phys. Rev. Lett. 48, 1636 (1982)
10. K.A. Olive, M.S. Turner: Phys. Rev. D 25, 213 (1982)
11. S. Colombi, S. Dodelson, L.M. Widrow: ApJ 458, 1 (1996)
12. J. Sommer-Larsen, A. Dolgov: astro-ph/9912166
13. C.J. Hogan: astro-ph/9912549, C.J. Hogan, J. Dalcanton: astro-ph/0002330
14. G.R. Blumenthal, S.M. Faber, J.R. Primack, M.J. Rees: Nature 311, 517 (1984)
15. N.A. Bahcall, J.P. Ostriker, S. Perlmutter, P.J. Steinhardt: Science 284, 1481 (1999)
16. M.S. Turner: ‘Cosmological Parameters’. In *Particle Physics and the Universe (Cosmo-98)*, ed. by D.O. Caldwell ((AIP Conf. Proc. 478, Woodbury, NY 1999)
17. J.R. Primack: ‘Status of Cosmology’. In *Cosmic Flows: Towards an Understanding of Large-Scale Structure*, ed. by S. Courteau, M.A. Strauss, J.A. Willick (ASP Conference Series), astro-ph/9912089 (1999)

18. S.S. Gershtein, Ya.B. Zel'dovich: JETP Lett. 4, 174 (1966)
19. G. Marx. A.S. Szalay: in *Neutrino '72* (Technoinform, Budapest 1971), vol. 1, p. 123
20. R. Cowsik, J. McClelland: Phys. Rev. Lett. 29, 669 (1972)
21. A.S. Szalay, G. Marx: A&A, 49, 437 (1976)
22. V.S. Lyubimov, E.G. Novikov, V.Z. Nozik, E.F., Tretyakov, V.S. Kosik: Phys. Lett. 94B, 266 (1980)
23. C. Caso et al.: Eur. Phys. J. C3, 1 (1998)
24. C. Weinheimer et al.: Phys. Lett. B 460, 219 (1999)
25. V.M. Lobashev et al.: Phys. Lett. B 460, 227 (1999)
26. S.M. Bilenky et al.: Phys. Lett. B, 465, 193 (1999); L. Baudis et al., Phys.Rev.Lett. 83, 41 (1999) ; cf. P. Fisher, B. Kayser, S. MacFarland, hep-ph/9906244 (1999); R.G.H. Robertson, hep-ex/0001034 (2000)
27. See <http://www.hep.anl.gov/NDK/Hypertext/nuindustry.html> — this is a comprehensive summary of neutrino data and ongoing experiments
28. S. Weinberg: *Gravitation and Cosmology* (Wiley, NY 1972)
29. D.J. Fixsen et al.: ApJ 473, 576 (1996)
30. B.W. Lee, S. Weinberg, Phys. Rev. Lett., 39, 165 (1977)
31. M.I. Vysotsky, A.D. Dolgov, Ya.B. Zeldovich: JETP Lett. 4, 120 (1977)
32. P. Hut, Phys. Lett. B69 85 (1977)
33. K. Sato, H. Kobayashi: Prog. Theor. Phys., 58, 1775 (1977)
34. J.E. Gunn, B.W. Lee, I. Lerche, D.N. Schramm, G. Steigman: Astrophys. J. 223, 1015 (1978)
35. H.-S. Kang, G. Steigman: Nucl. Phys. B 372, 494 (1992)
36. S.D. Tremain, J.E. Gunn: Phys. Rev. Lett. 42, 407 (1979)
37. S.M. Faber, D.N.C. Lin: Astrophys. J. 266, L17 (1983)
38. C. Pryor, J. Kormendy: Astron. J. 100, 127 (1990)
39. D.N.C. Lin, S.M. Faber: ApJ 266, L21 (1983)
40. O.E. Gerhard, D.N. Spergel: ApJ 389, L9 (1992)
41. D.N. Spergel, D.H. Weinberg, J.R. Gott: Phys. Rev. D. 38, 2014 (1988)
42. G.S.Bisnovatyi-Kogan, I.D. Novikov: Sov. Astron. 24, 516 (1980)
43. J.R. Bond, G. Efstathiou, J. Silk: Phys. Rev. Lett 45, 1980 (1980)
44. J.R. Bond, G. Efstathiou: ApJ 285, L45 (1984)
45. A. Melott: Mon. Not. Roy. Astr. Soc. 202, 595 (1983)
46. J. Centrella, A. Melott: Nature, 305, 196 (1983)
47. A.A. Klypin, S.F. Shandarin: Mon. Not. Roy. Astr. Soc. 204, 891 (1983)
48. C. Frenk, S.D.M. White, M. Davis: ApJ 271, 417 (1983)
49. Ya.B. Zel'dovich, J. Einasto, S.F. Shandarin: Nature, 300, 407 (1982)
50. A. Dekel, S.J. Aarseth: Astrophys. J., 283, 1 (1984)
51. N. Kaiser: Astrophys. J. 273, L17 (1983)
52. P.R. Shapiro, C. Struck-Marcell, A.L. Melott: Astrophys. J. 275, 413 (1983)
53. S.D.M. White: in *Inner Space/Outer Space*, ed. by E.W. Kolb, et al. (Univ. of Chicago Press, Chicago 1986).
54. J.R. Bond, A.S. Szalay, S.D.M. White: Nature 301, 584 (1983)
55. G. Smoot et al.: Astrophys. J. 396, L1 (1992)
56. U.-L. Pen, U. Seljak, N. Turok: Phys. Rev. Lett. 79, 1611 (1997)
57. A. Albrecht, R.A. Battye, J. Robinson: Phys. Rev. D 59, 023508, 1 (1999)
58. W. Hu, D. Eisenstein, M. Tegmark: Phys. Rev. Lett. 80, 5255 (1998)
59. D. Brodbeck, D. Hellinger, R. Nolthenius, J.R. Primack, A.A. Klypin: ApJ 495, 1 with accompanying 25 minute videotape (1998)

60. C.-P. Ma: ‘Neutrinos and Dark Matter’. In *Neutrinos in Physics and Astrophysics: from 10^{-33} to 10^{+28} cm*, ed. by P. Langacker (World Scientific, Singapore), astro-ph/9904001
61. J. Holtzman: ApJ Suppl. 71, 1 (1989)
62. A.A. Klypin, J. Holtzman: astro-ph/9712217 (1997)
63. J. Holtzman, J.R. Primack: ApJ 405, 428 (1993)
64. U. Seljak, M. Zaldarriaga: ApJ 469, 437 (1996)
65. C.-P. Ma, E. Bertschinger: ApJ 455, 7 (1995)
66. C.-P. Ma: ApJ 471, 13 (1996)
67. D. Eisenstein, W. Hu: ApJ 511, 5 (1998)
68. A.A. Klypin, J. Holtzman, J.R. Primack, E. Regös), ApJ 416, 1 (1993)
69. R.K. Schaefer, Q. Shafi: Phys. Rev. D 49, 4990 (1994)
70. R. Nolthenius, A.A. Klypin, J.R. Primack: ApJ 422, L45 (1994)
71. S. Ghigna, S. Borgani, S. Bonometto, L. Guzzo, A. Klypin, J.R. Primack, R. Giovanelli, M. Haynes: ApJ, 437, L71 (1994)
72. G. Kauffmann, S. Charlot: ApJ 430, L97 (1994)
73. J. Miralda-Escude, H.-J. Mo: ApJ 430, L25 (1994)
74. C.-P. Ma, E. Bertschinger: ApJ 434, L5 (1994)
75. A.A. Klypin, S. Borgani, J. Holtzman, J.R. Primack: ApJ 444, 1 (1995)
76. A.A. Klypin, R. Nolthenius, J.R. Primack: ApJ 474, 533 (1997)
77. S. Ghigna, S. Borgani, M. Tucci, S.A. Bonometto, A.A. Klypin, J.R. Primack: ApJ 479, 580 (1997)
78. R. Nolthenius, A.A. Klypin, J.R. Primack: ApJ 480, 43 (1997)
79. C.-P. Ma et al.: ApJ 484, L1
80. M.G. Haehnelt, M. Steinmetz, M. Rauch: ApJ 495, 647 (1998)
81. J.X. Prochaska, A.M. Wolfe: ApJ 487, 73 (1997)
82. D. Pogosyan, A. Starobinsky: astro-ph/9502019
83. Y. Fukuda et al.: Phys. Lett B 335, 237 (1994)
84. Y. Fukuda et al.: Phys. Rev. Lett. 81, 1562 (1998)
85. M. Apollonio et al.: Phys. Lett. B 420, 397 (1998)
86. X. Shi, D.N. Schramm, & B.D. Fields: Phys. Rev. D 48, 2563 (1993); C.Y. Cardall, G.M. Fuller Phys Rev D 54, 1260 (1996); T.I Izotov, T.X. Thuan: ApJ 500, 188 (1998)
87. C. Athanassopoulos et al.: Phys. Rev. Lett. 77, 3082 (1996); Phys. Rev. C 54, 2685 (1996)
88. C. Athanassopoulos et al. Phys. Rev. Lett. 81, 1774 (1998)
89. M. Fukugita, C.J. Hogan, P.J.E. Peebles: ApJ 503, 518 (1998)
90. See, e.g., W.C. Haxton: Ann. Rev. Astro. Astroph. 33, 459 (1995); V. Castellani et al.: Phys. Rep. 281, 309 (1997)
91. S.L. Glashow, P.J. Kernan, L.M. Krauss: Phys. Lett. B 445, 412 (1999); cf. F. Vissani: hep-ph/9708483, and H. Georgi, S.L. Glashow 1998, hep-ph/9808293 (1998)
92. D.O. Caldwell, R.N. Mohapatra: Phys. Rev. D 48, 3259 (1993), Phys. Rev. D 50, 3477 (1994); J.T. Peltoniemi, J.W.F. Valle: Nucl. Phys. B 406, 409 (1993); V. Barger, Weiler, T.J., Whisnant, K.: Phys. Lett. B 427, 97 (1998); S.C. Gibbons et al.: Phys. Lett. B 430, 296 (1998)
93. D.O. Caldwell: in 23rd Johns Hopkins Workshop, *Neutrinos in the Next Millennium*, hep-ph/9910349 (1999); D.O. Caldwell, G.M. Fuller, Y.-Z. Qian: astro-ph/9910175 (1999); cf. G.M. Fuller, J.R. Primack, Y.-Z. Qian: Phys. Rev. D 52, 1288 (1995)

94. A. Zee: Phys. Lett. 93B, 389 (1980), Phys. Lett. B161, 141 (1985); cf. A. Smirnov: hep-ph/9611465 (1996)
95. K.S. Babu, J.C. Pati, F. Wilczek: Nucl. Phys. B 566, 33 (2000)
96. P.M. Garnavich et al.: ApJ 493, L53 (1998), 509, 74 (1998); S. Perlmutter et al.: ApJ 517, 565 (1999)
97. R.S. Somerville, J.R. Primack, S.M. Faber 1998, astro-ph/9806228, MNRAS in press
98. S. Burles, D. Tytler: ApJ 499, 699 (1998); Space Sci. Rev. 84, 65 (1998)
99. A.E. Evrard: MNRAS 292, 289 (1997)
100. J.E. Carlstrom: astro-ph/9905255 (1999)
101. V.R. Eke, S. Cole, C.S. Frenk, J.P. Henry: MNRAS 298, 1145 (1998)
102. J.P. Henry: ApJ, in press, astro-ph/0002365 (2000)
103. C.C. Smith, A.A. Klypin, M.A.K. Gross, J.R. Primack, J. Holtzman: MNRAS 297, 910 (1998)
104. C.B. Netterfield et al.: ApJ 474, 47 (1997)
105. P.F.S. Scott et al.: ApJ 461, L1 (1996)
106. J.C. Baker 1997: in *Proceedings of the Particle Physics and the Early Universe Conference*, www.mrao.com.ac.uk/ppauc/proceedings
107. A.D. Miller et al.: ApJ, 524, L1 (1999) [we used D1+D2 data]
108. C. Baugh, G. Efstathiou: MNRAS 265, 145 (1993)
109. M. Girardi, S. Borgani, G. Giuricin, F. Mardirossian, M. Mezzetti: ApJ 506, 45 (1998)
110. M.A.K. Gross, R.S. Somerville, J.R. Primack, S. Borgani, M. Girardi: in *Large Scale Structure: Tracks and Traces*, ed. by V. Müller, S. Gottlöber, J.P. Mücke, J. Wambsganss (World Scientific, Singapore 1998)
111. R. Valdarnini, T. Kahniashvili, B. Novosyadlyj 1998, A&A 336, 11
112. M. Fukugita, G.-C. Lin, N. Sugiyama: Phys. Rev. Lett. 84, 1082 (2000)
113. S.A. Bonometto, E. Pierpaoli: New Astron. 3, 391 (1998)
114. J.R. Primack, M.A.K. Gross: 'Cold + Hot Dark Matter After Super-Kamiokande'. In *The Birth of Galaxies*, ed. by B. Guiderdoni (Editions Frontières, 2000), astro-ph/9810204
115. J.R. Primack, M.A.K. Gross: 'Hot Dark Matter in Cosmology'. astro-ph (2000)
116. R.A.C. Croft, W. Hu, R. Davé: Phys. Rev. Lett. 83, 1092 (1999)
117. A.A. Klypin, J.R. Primack, J. Holtzman: ApJ 466, 1 (1996)
118. A. Jenkins et al.: ApJ 499, 20 (1998)
119. P. Colin, A.A. Klypin, A. Kravtsov, A. Khokhlov: ApJ 523, 32 (1999)
120. R. Davé, D. Hellinger, J.R. Primack, R. Nolthenius, A.A. Klypin: MNRAS 284, 607 (1997)
121. D. Eisenstein, M. Zaldarriaga: astro-ph/9912149 (1999)
122. E. Carretta, R.G. Gratton, G. Clementini, & F. Fusi Pecci: astro-ph/9902086 (1999)
123. A.R. Cooray: A&A, 348, 31 (1999)