

## Cosmological Imprint of an Energy Component with General Equation of State

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(Received 28 July 1997)

We examine the possibility that a significant component of the energy density of the Universe has an equation of state different from that of matter, radiation, or cosmological constant ( $\Lambda$ ). An example is a cosmic scalar field evolving in a potential, but our treatment is more general. Including this component alters cosmic evolution in a way that fits current observations well. Unlike  $\Lambda$ , it evolves dynamically and develops fluctuations, leaving a distinctive imprint on the microwave background anisotropy and mass power spectrum. [S0031-9007(98)0521-5]

PACS numbers: 98.80.Cq, 95.35.+d, 98.65.Dx, 98.70.Vc

Inflationary cosmology predicts that the Universe is spatially flat and that the total energy density of the Universe is equal to the critical density. This prediction is consistent with current measurements of the cosmic microwave background (CMB) anisotropy and may be verified with high precision in the next generation of CMB satellite experiments. At the same time, there is growing observational evidence that the total matter density of the Universe is significantly less than the critical density [1]. If this latter result holds and the CMB anisotropy establishes that the Universe is flat, then there must be another contribution to the energy density of the Universe. One candidate that is often considered is a cosmological constant  $\Lambda$  or vacuum energy density. The vacuum density is a spatially uniform, time-independent component. Cold dark matter (CDM) models with a substantial cosmological constant ( $\Lambda$ CDM) are among the models which best fit existing observational data [1]. However, it should be emphasized that the fit depends primarily on the fact that the models have low matter density and are spatially flat; the fit is not a sensitive test of whether the additional energy contribution is vacuum energy.

In this paper, we consider replacing  $\Lambda$  with a dynamical, time-dependent, and spatially inhomogeneous component whose equation of state is different from baryons, neutrinos, dark matter, or radiation. The equation of state of the new component, denoted as  $w$ , is the ratio of its pressure to its energy density. This fifth contribution to the cosmic energy density, referred to here as “quintessence” or  $Q$  component, is broadly defined, allowing a spectrum of possibilities including an equation of state which is constant, uniformly evolving or oscillatory. Examples of a  $Q$  component are fundamental fields (scalar, vector, or tensor) or macroscopic objects, such as a network of light, tangled cosmic strings [2]. The analysis in the present paper applies to any component whose hydrodynamic properties can be mimicked by a scalar field evolving in a potential which couples to matter only through gravitation. In particular, we focus on equations of state with  $-1 < w < 0$  because this range fits current cosmological observations best [3–7]. This has motivated several investigations [3,4,8,9] of components with

$w < 0$  in which a spatially uniform distribution has been assumed, e.g., a decaying  $\Lambda$  or smooth component.

In this Letter, we begin by arguing that a smoothly distributed, time-varying component is unphysical—it violates the equivalence principle. Hence, predictions of CMB and mass power spectra which have not included fluctuations in the new component are not valid. We outline the general conditions needed to have  $w < 0$  consistent with the equivalence principle and stable against catastrophic gravitational collapse. We show that an evolving scalar field automatically satisfies these conditions. We then compute the CMB and mass power spectra for a wide, representative class of models [see Figs. 1 and 2(a)], taking careful note of the effects of the fluctuations in the  $Q$  component. We show that the fluctuations leave a distinctive signature that enables a  $Q$  component to be distinguished from dark matter and a cosmological constant and makes it possible to resolve its equation of state. Comparing to observations of the CMB and large-scale structure [Fig. 2(b)], we identify a new spectrum of plausible models that will be targets for future experiments.

Introducing a dynamical energy component is at least as well motivated by fundamental physics as introducing a cosmological constant. In fact, the theoretical prejudice based on fundamental physics is that  $\Lambda$  is precisely zero; if it is nonzero, there is no conceivable mechanism to explain why the vacuum density should be comparable to the present matter density, other than arguments based on the anthropic principle. On the other hand, dynamical fields abound in quantum gravity, supergravity, and superstring models (e.g., hidden sector fields, moduli, pseudo-Nambu-Goldstone bosons), and it may even be possible to utilize the interaction of these fields with matter to find a natural explanation why the  $Q$  component and matter have comparable energy densities today.

As noted above, a number of studies [3,4,8] have assumed a “smooth” (spatially uniform), time-dependent component with an arbitrary equation of state (sometimes called  $x$ CDM) which does not respond to the inhomogeneities in the dark matter and baryon-photon-neutrino fluid. In computing the CMB anisotropy, one finds a near

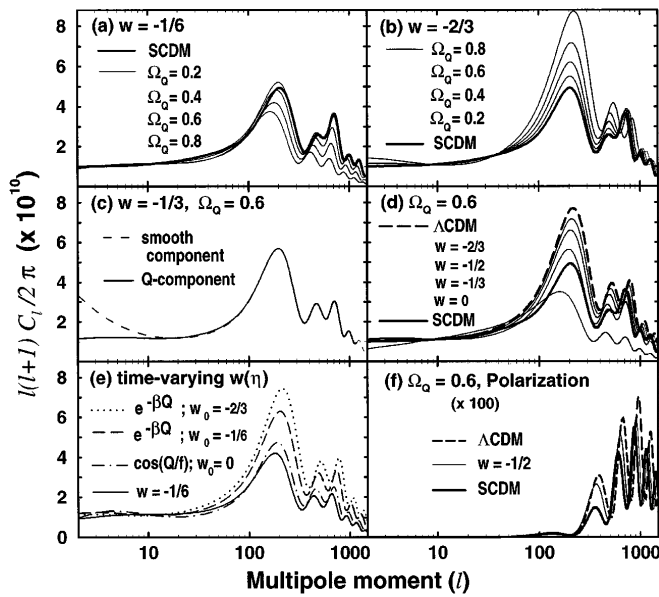


FIG. 1. CMB power spectrum,  $\ell(\ell + 1)C_\ell/2\pi$  vs  $\ell$  where  $C_\ell$  is the multipole moment, illustrating that the spectrum changes significantly as a function of  $w$  and  $\Omega_Q$ . Thick solid lines represent SCDM; thick dashed lines correspond to  $\Lambda$ CDM. For thin solid lines, legends list the parameters in sequence according to the height of the curve beginning from the topmost. The variation with  $\Omega_Q$  is shown in (a) and (b); (c) compares the predictions for a  $Q$  component with fluctuations ( $\delta Q \neq 0$ ) to a smooth component ( $\delta Q = 0$ ). The differences due to  $\delta Q$  at large angular scales change COBE normalization which is responsible for the substantial variation of the acoustic peaks with  $w$ , as shown in (d). Panel (e) shows the results for time-varying  $w(\eta)$  as determined by specifying a potential  $V(Q)$  and initial conditions, where  $V(Q)$  is an exponential or cosine functional of  $Q$  in the examples shown. Included also is the prediction for a constant  $w = -1/6$ , which differs from the results where  $w(\eta)$  is time varying and  $w(\eta_0) = -1/6$ . In (f) is shown the polarization (amplified 100-fold) for  $w = -1/2$  compared to SCDM and  $\Lambda$ CDM.

degeneracy [3] with  $\Lambda$ CDM for a wide range of  $w$ . In this Letter, we show that the degeneracy is significantly broken when fluctuations in the  $Q$  component are included. However, it is important to realize that the smooth and  $Q$  scenarios are not competing models. A smooth, time-evolving component is ill defined, since the smoothness is gauge dependent, and unphysical, because it violates the equivalence principle to ignore the response of the new component to the inhomogeneities in the surrounding cosmological fluid. Hence, a fluctuating, inhomogeneous component is the only valid way of introducing an additional energy component.

There have been some discussions in the literature of an energy component consisting of a dynamical, fluctuating cosmic scalar field evolving in a potential [5,10,11], or an energy component evolving according to a specific equation of state [4,8,12]. In the case of a scalar field, the CMB anisotropy and power spectrum were computed by Coble *et al.* for a cosine potential [5], and by Ferreira and Joyce for an exponential potential [11]. Here we go beyond these isolated examples to explore the range of

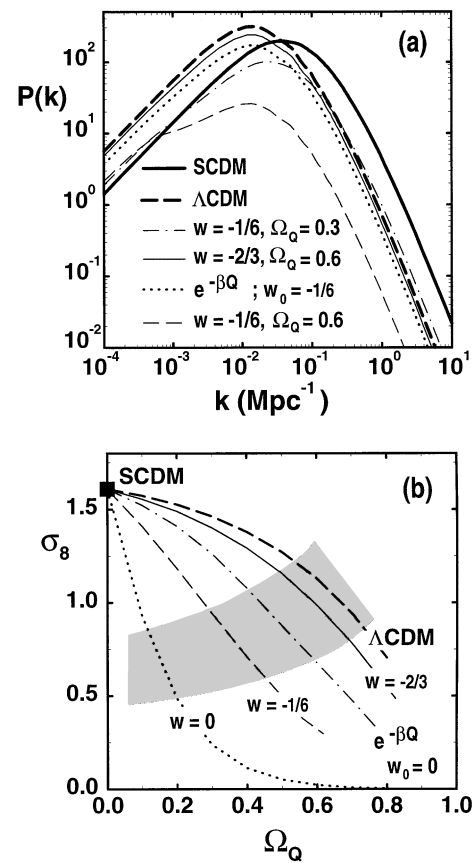


FIG. 2. (a) Variation of mass power spectrum for some representative QCDM examples. (b) The variation of  $\sigma_8$  with  $\Omega_Q$ . For  $\Lambda$ CDM,  $\Omega_Q$  is  $\Omega_\Lambda$ . The suppression of  $\sigma_8$  in QCDM compared to SCDM makes for a better fit with current observations. The gray swath illustrates constraints from x-ray cluster abundance.

possibilities and the range of imprints on the CMB and large-scale structure.

The class of cosmological models we consider in this work are spatially flat, Friedmann-Robertson-Walker (FRW) space-times which contain baryons, neutrinos, radiation, cold dark matter, and the  $Q$  component (QCDM models). The space-time metric is given by  $ds^2 = a^2(\eta)(-d\eta^2 + dx^2)$  where  $a$  is the expansion scale factor and  $\eta$  is the conformal time. For the purposes of CMB and mass power spectrum prediction, we model the  $Q$  component of the fluid as a scalar field  $Q$ , with self-interactions determined by a potential  $V(Q)$ . In an ideal adiabatic fluid with  $w < 0$ , a concern would be that the sound speed is imaginary,  $c_s^2 = w < 0$ , and small wavelength perturbations are hydrodynamically unstable [13]. However, a scalar field is not an ideal fluid in this sense. The sound speed is a function of wavelength, increasing from negative values for long wavelength perturbations and approaching  $c_s^2 = 1$  at short wavelengths (shorter than the horizon size) [14]. Consequently, for the cases considered here, small wavelength modes remain stable. In general, our treatment relies only on properties of  $w$  and  $c_s^2$ , and so it applies both to cosmic scalar fields and

to any other forms of matter-energy (e.g., nonscalar fields or topological defects) with  $w$  and  $c_s^2$  which can also be obtained by a scalar field and some potential. Otherwise, we make no special assumptions about the microscopic composition of the  $Q$  component.

We implicitly assume that any couplings to other fields are negligibly small, so that the scalar field interacts with other matter only gravitationally. The average energy density is  $\rho_Q = \frac{1}{2a^2} Q'^2 + V$  and the average pressure is  $p_Q = \frac{1}{2a^2} Q'^2 - V$ , where the prime represents  $\partial/\partial\eta$ . As  $Q$  evolves down its potential, the ratio of kinetic ( $\frac{1}{2a^2} Q'^2$ ) to potential ( $V$ ) energy can change; this would lead to a time-varying equation of state  $w \equiv p_Q/\rho_Q$ . In this paper, we consider constant and time-varying  $w$  models for which  $w \equiv p_Q/\rho_Q \in [-1, 0]$ , since this range includes models which best fit current observations.

We have modified numerical Boltzmann codes for computing CMB anisotropy and the mass power spectrum. In one approach, we use an *explicitly defined equation of state*  $w(\eta)$ , which spans all models but requires that the equation of state as a function of time be known. All properties of the model are determined by the proscribed  $w(\eta) \equiv w[a(\eta)]$  and

$$\rho_Q = \Omega_Q \rho_o \exp\left[3 \ln \frac{a_o}{a} + 3 \int_a^{a_o} \frac{da}{a} w(a)\right], \quad (1)$$

where  $\rho_o$  and  $a_o$  are, respectively, the critical energy density and scale factor today. An effective scalar potential which produces this  $\rho_Q$  is computed as a by-product. A second approach uses an *implicitly defined equation of state* in which we specify the potential  $V(Q)$  directly, which is useful for exploring specific models motivated by particle physics.

A series of codes has been developed for synchronous and for conformal Newtonian gauge by modifying standard algorithms [15–17]. In the synchronous gauge, the line element is given by [18]

$$ds^2 = a^2(\eta)[-d\eta^2 + (\gamma_{ij} + h_{ij})dx^i dx^j], \quad (2)$$

where  $\gamma_{ij}$  is the unperturbed spatial metric, and  $h_{ij}$  is the metric perturbation. The scalar field fluctuation  $\delta Q$  obeys the equation

$$\delta Q'' + 2 \frac{a'}{a} \delta Q' - \nabla^2 \delta Q + a^2 V_{,QQ} \delta Q = -\frac{1}{2} h' Q'. \quad (3)$$

Here,  $h$  is the trace of the spatial metric perturbation, as described by Ma and Bertschinger [16].

We have found the observable fluctuation spectrum to be insensitive to a broad range of initial conditions, including the case in which the amplitudes of  $\delta Q$ ,  $\delta Q'$  were set by inflation. All of the examples shown in this paper have  $\delta Q = \delta Q' = 0$  initially as measured in synchronous gauge. With this choice of initial conditions, the imprint of  $\delta Q$  on the CMB anisotropy would be negligible if we did not include the response of  $Q$  to the matter density perturbations through the  $h' Q'$  source term in Eq. (3). If

$Q' = 0$ , as occurs for  $\Lambda$ CDM, the scalar field remains unperturbed. In the implicit approach where  $V$  is given, the energy density, pressure, and momentum perturbations are

$$\begin{aligned} \delta \rho_Q &= \frac{1}{a^2} Q' \delta Q' + V_{,Q} \delta Q, \\ \delta p_Q &= \frac{1}{a^2} Q' \delta Q' - V_{,Q} \delta Q, \\ (\rho_Q + p_Q)(v_Q)_i &= -\frac{1}{a^2} Q' (\delta Q)_{,i}. \end{aligned} \quad (4)$$

These quantities must be included in the evolution of  $h$ , the total fluctuation density contrast  $\delta$ , and the total velocity perturbation  $v$ . For cases where  $w(\eta)$  is provided explicitly, the same relations may be applied where an effective  $V$  is computed by the code from  $w(\eta)$  [18].

Although we have examined many models [18], we restrict ourselves here to a representative spectrum. Figure 1 illustrates CMB studies showing Cosmic Background Explorer (COBE)-normalized [19] CMB anisotropy spectra assuming adiabatic initial conditions in the matter with tilt  $n = 1$ , Hubble constant  $H_0 = 100h \text{ km s}^{-1} \text{ Mpc}^{-1}$  with  $h = 0.65$ , and  $\Omega_b h^2 = 0.02$ , where  $\Omega_b$  is the ratio of the baryon density to the critical density. Because the models are spatially flat, the angular size of the sound horizon at last scattering is nearly the same for all our models and, consequently, the acoustic oscillation peaks are at nearly the same  $\ell$ 's as in standard CDM (SCDM) [20,21]. However, the shape of the plateau for low  $\ell$  and the shapes of the acoustic peaks at high  $\ell$  are distorted by three effects: a combination of early and late integrated Sachs-Wolfe effects [21] due to having a fluid component with  $w \neq 0$  and the direct contribution of  $\delta \rho_Q$  on the CMB. Figures 1(a) and 1(b) show how the spectra vary with  $\Omega_Q$ , the ratio of the energy density in  $Q$  to the critical density, for fixed  $w$ . The behavior is different for different equations of state. For  $w = -1/6$  the first acoustic peak rises and then steadily decreases as  $\Omega_Q$  increases; for  $w = -2/3$ , the acoustic peak rises uniformly as  $\Omega_Q$  increases. Figure 1(c) compares predictions assuming a smooth, nonfluctuating component (a case we argued is unphysical) with the full spectrum including fluctuations in  $Q$ , illustrating the substantial difference at large angular scales. Figure 1(d) shows spectra for fixed  $\Omega_Q = 0.6$  and varying  $w$ . When fluctuations in  $Q$  are not included, the curves for  $-1 < w < -1/2$  are very nearly degenerate [3], but the degeneracy is substantially broken in Fig. 1(d) by the contribution of  $Q$  fluctuations at large angular scales at a level much greater than cosmic variance. Figure 1(e) illustrates results for a time-varying  $w(\eta)$  obtained using exponential [10] and cosine [5] scalar potentials. The exponential potentials are of the form  $V(Q) = m^4 \exp[-\beta Q]$  where  $Q = Q' = 0$  initially; for the examples shown,  $(m, \beta) = (0.00358 \text{ eV}, 11.66 m_p^{-1})$  for the case with  $w(\eta_0) = -1/6$  and  $(m, \beta) = (0.00243 \text{ eV}, 8.0 m_p^{-1})$  for

$w(\eta_0) = -2/3$  as expressed in Planck mass ( $m_p$ ) units. For the cosine potential,  $V(Q) = m^4(1 + \cos[Q/f])$ , where  $(m, f) = (0.00465 \text{ eV}, 0.1544m_p)$  with  $Q = 1.6f$  and  $Q' = 0$  initially [5]. Two of the curves are especially interesting because they compare the CMB anisotropy for two models with the same value of  $w$  today but with different values of  $w$  in the past. One model has constant  $w = -1/6$ , whereas the other model, based on a scalar field rolling down an exponential potential, has  $w$  evolving from  $-1$  to  $-1/6$ . The two models produce easily distinguished CMB power spectra even though the equation of state is the same today due to the difference in past evolutionary history and the direct contributions of  $\delta Q$ . Hence, the evolution of  $w$  can also be determined from CMB measurements. Also shown are examples of other exponential and cosine potentials compared with  $\Lambda$ CDM which illustrate that the CMB anisotropy is sensitive to  $V(Q)$ . Figure 1(f) illustrates an example of CMB polarization, which has similar variations with parameters as the temperature anisotropy.

Figure 2 illustrates the mass power spectrum predictions. The key feature to note in Fig. 2(a) is that the power spectrum is sensitive to all parameters: the value of  $w$ ; the time-dependence of  $w$ ; the effective potential  $V(Q)$  and initial conditions; and the value of  $\Omega_Q$ . Hence, combined with the CMB anisotropy, the power spectrum provides a powerful test of QCDM and its parameters. An important effect is the suppression of the mass power spectrum and  $\sigma_8$  (the rms mass fluctuation at  $8h^{-1}$  Mpc) compared to  $\Lambda$ CDM, which makes for a better fit to current observations [1]. The gray swath in Fig. 2(b) represents the constraint from x-ray cluster abundance for  $\Lambda$ CDM [22]; in general, the constraint is weakly model dependent.

While QCDM and  $\Lambda$ CDM both compare well to current observations of CMB and of large-scale structure today, QCDM has advantages in fitting constraints from high redshift supernovas, gravitational lensing, and structure formation at large redshift ( $z \approx 5$ ). Constraints based on classical cosmological tests on  $\Lambda$  ( $w_\Lambda = -1$ ), such as supernovas and lensing, are significantly relaxed for QCDM with  $w \approx -1/2$  or greater [23]. Another property of QCDM (or  $\Lambda$ CDM) is that structure growth and evolution ceases when the  $Q$  component (or  $\Lambda$ ) begins to dominate over the matter density. Comparing QCDM and  $\Lambda$ CDM models with  $\Omega_Q = \Omega_\Lambda$ , this cessation of growth occurs earlier in QCDM. For larger values of  $w$ , the growth ceases earlier. Hence, more large-scale structure and quasar formation at large redshift is predicted, in better accord with deep redshift images.

In conclusion, we find that the ‘‘quintessence’’ hypothesis fits all current observations and results in an imprint on the CMB anisotropy and mass power spectrum that should be detectable in near-future experiments. Its discovery could indicate the existence of new, fundamental fields with profound implications for particle physics, as well as cosmology. A number of follow-up studies are underway including: a quantitative analysis to determine

how well future CMB experiments can resolve a  $Q$  component and how well one can simultaneously resolve other cosmic parameters; numerical simulations of galaxy formation and evolution in the presence of a  $Q$  component; and search for quintessential candidates in models of fundamental physics.

We thank P. J. E. Peebles, J. P. Ostriker, and M. White for useful comments. This research was supported by the Department of Energy at the University of Pennsylvania, Report No. DE-FG02-95ER40893.

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