Ph 608 Problem Set 2

Due: Monday, April 3, 2006

1. Peacock problem 5.2

2. Peacock problem 9.3

3. In supersymmetry models where the gluino mass and explicit supersymmetry-breaking masses in the neutralino mass matrix are all zero, the gluino $\tilde{g}$ and photino $\tilde{\gamma}$ acquire small masses through radiative corrections. Below a critical temperature, the gluinos condense into R^0 bosons composed of a gluino and an antigluino, with mass $Mc^2 = 1$ to 2 GeV, while the photino mass is $mc^2 = 100$ to 1400 MeV. This light photino is a dark matter candidate.

The following five reactions are important for the evolution of the photino abundance:

R^0 self-annihilation, $R^0 R^0 \to X$ with the (thermally-averaged) cross section

$$\langle |v| \sigma_{RR} \rangle = 600 x^{-1} r^{-1}\,\text{mb};$$

$\tilde{\gamma}$ self-annihilation, $\tilde{\gamma}\tilde{\gamma} \to X$, with cross section

$$\langle |v| \sigma_{\gamma\gamma} \rangle = 2.0 \times 10^{-11} x^{-1}\,[\mu_8^2 \mu_S^{-4}]\,\text{mb};$$

coannihilation, $\tilde{\gamma} R^0 \to X$, with cross section

$$\langle |v| \sigma_{\gamma R} \rangle = 1.5 \times 10^{-10} r\,[\mu_8^2 \mu_S^{-4} A]\,\text{mb};$$

decay of the R^0, $R_0 \to \tilde{\gamma}\pi$ with decay width

$$\Gamma_R = 2.0 \times 10^{-14} r^5 (1 - r^{-1})^6\,[\mu_8^5 \mu_S^{-4} B]\,\text{GeV};$$

and interconversion of $\tilde{\gamma}$ and R^0, $R^0\pi \leftrightarrow \tilde{\gamma}\pi$, with approximate cross section for either process

$$\langle |v| \sigma_{\gamma R} \rangle = 1.5 \times 10^{-10} r\,[\mu_8^2 \mu_S^{-4} C]\,\text{mb}.$$

These expressions use the abbreviations $x \equiv (mc^2)/(kT)$, $r \equiv M/m$, and $\mu_8 \equiv mc^2/(800\,\text{GeV})$. The squark mass scale in units of 100 GeV is given by μ_S, and A, B, and C are coefficients reflecting uncertainties involving the calculation of hadronic matrix elements. Assume that $A = B = C = \mu_S = 1$.

The goal of this problem is to determine the relic abundance of light photinos, assuming that all particles are in thermal equilibrium until their reactions freeze out (*i.e.*, until the reaction rate per particle equals the Hubble expansion rate).

a. Show that, when the universe is radiation dominated,

$$H = 1.66 g_*^{1/2} \frac{(kT)^2}{m_{pl} c^2 \hbar},$$

where the Planck mass $m_{pl} = \sqrt{\hbar c/G}$. Use this expression to write H as a function of x and μ_8, assuming that $g_* = 100$ in the relevant temperature range.

b. Use the equilibrium Maxwell-Boltzmann number densities of the R^0 and $\tilde{\gamma}$ to write the reaction rates per particle as a function of x and r.

c. Use the above reaction rates to argue that the R^0 stays in equilibrium until well after the photino has frozen out.

d. Write a small program that calculates the freeze-out temperature given by each of the four photino reaction rates for a given R^0 and photino mass. The lowest of these freeze-out temperatures determines the actual final photino number density. Have your program calculate the $\Omega_{\tilde{\gamma}} h^2$ today implied by this number density.

e. Calculate $\Omega_{\tilde{\gamma}} h^2$ on a grid of (r, μ_8) values covering the range given at the beginning of this problem. For what values is $\Omega_{\tilde{\gamma}} h^2 \simeq 0.13$, making the light photino an interesting dark matter candidate? Show that for these values, the interconversion reaction determines the photino freeze-out rather than photino self-annihilations.