Ph 442 Problem Set 5

Due: Thursday, October 29, 2009

1. If the cooling behind the shock front is very effective, the gas temperature returns quickly to its pre-shock value. If we are not interested in the details of the cooling region, we can regard it and the shock itself as a single discontinuity, usually called an isothermal shock, and can treat it in the manner used for non-radiating shocks. Thus, we can retain the mass and momentum jump conditions but replace the energy condition by $T_0 = T_1$, where T_0 and T_1 are the gas temperatures on each side of the discontinuity or, equivalently, by the equality of sound speeds: $c_0 = c_1 = c$. Here $c_0 = (P_0/\rho_0)^{1/2}$ and so on.

a. Show that

$$v_1 = \frac{c}{\mathcal{M}_0} \tag{1}$$

$$\rho_1 = \mathcal{M}_0^2 \rho_0. \tag{2}$$

Here $\mathcal{M}_0$ is the preshock Mach number. Note that strong isothermal shocks result in very large compressions.

b. Also show that

$$P_1 = \rho_0 v_0^2. \tag{3}$$

Thus, the post-shock gas pressure is equal to the pre-shock ram pressure.

c. What is the number density behind the shock, n_1, for $n_0 = 1$ atom cm^{-3}, $c_0 = 3$ km s^{-1}, and a shock speed of 50 km s^{-1}? If the shock maintains this speed, estimate the thickness of the shell of swept-up material that accumulates behind the shock in 10^5 yrs. Assume a planar shock for simplicity.

2. Consider a non-radiating shock expanding into an interstellar medium composed of neutral hydrogen with a number density of $n_0 = 10$ atoms cm^{-3} and a temperature of $T_0 = 10^2$ K.

a. Calculate the sound speed in the unperturbed interstellar medium.

b. Calculate the temperature of the gas behind the shock for shock speeds of 5×10^3, 500, and 50 km s^{-1}. Assume a strong shock and an adiabatic index of 5/3 for the gas. This ignores the energy needed to ionize the hydrogen, but this is an OK approximation for the temperatures involved.

c. Calculate the characteristic cooling timescale (*i.e.*, $e/(de/dt)$, where e is the thermal energy density) for the gas behind the shock for the three shock velocities of part (b). Use the plot of the energy emitted by a hot plasma as a function of temperature given on the last page of the Garmire notes, or the formula for power emitted per unit volume by thermal bremsstrahlung given in those notes or in class for temperatures higher than the plot, or the attached cooling curve for temperatures below 10^5 K. Note that you will need to determine the density behind the shock, again assuming that the shock is strong. Both plots show the energy loss rate divided by the square of a number density.

d. For which of the three shock velocities above is the assumption of a non-radiating shock consistent with your calculated cooling time, assuming that the shock is associated with a supernova remnant?

3. Charge particle (cosmic ray) dynamics.

a. Calculate the energy of a cosmic ray proton (in eV) which has a circular orbit radius equal to the radius of the Galaxy. Assume a magnetic field strength of 10^{-6} gauss (the gauss is the cgs unit of magnetic field strength) and a radius for the Galaxy of 10 kpc. Also assume that all of the velocity of the proton is perpendicular to the magnetic field lines.

b. The Auger Observatory found that cosmic ray protons with energies of greater than about 6×10^{19} eV seemed to be preferentially coming from directions within about $3°$ of locations of active galactic nuclei on the sky. Estimate the angle by which a cosmic ray proton will be deflected by a Galactic magnetic field of 10^{-6} gauss as it traverses a distance of about 10 kpc through our Galaxy. Adopt an average energy of 8×10^{19} eV for the cosmic rays. How does your value compare with the angle of $3°$?

c. It is inferred that electrons of energy 10^{14} eV (100 TeV) are responsible for emitting the synchrotron X-rays that are observed from some supernova remnants, in particular SN 1006. Compare the radius of the orbit of these electrons with the size of the supernova remnant. Assume that $B = 10^{-5}$ gauss within the remnant.

4. Use the sequence of images of the Crab supernova remnant taken by the Chandra X-ray observatory, available at http://chandra.harvard.edu/photo/2002/0052/index.html and linked on the class homepage, to estimate the velocity with which plasma is expanding away from the pulsar at the middle of the nebula. Include a sketch or image to show which feature you tracked. Take the distance to the Crab to be 2000 pc.