Ph 442 Problem Set 2

Due: Thursday, September 24, 2009

1. Whether the collapse of the core of a massive star results in a supernovae depends on what happens in the region below the standing shock that forms after the core bounce. This problem examines some of what is happening in the "gain region" immediately below the shock where the energy gain from neutrino absorption is larger than the energy loss from neutrino emission.

a. A simple model for the gain region is that the matter emerges from the shock with a constant inward velocity, v_g, which it maintains through the gain region. Assume that this region extends from the radius of the shock, r_s, to a radius of $r_s/2$. Calculate the time for matter to cross the gain region for $r_s = 150$ km and $v_g = 5000$ km s^{-1}. Make a *simple* estimate of the time for matter starting at rest at r_s to fall to zero radius due to the gravitational acceleration of 1 M$_\odot$ in the dense collapsed core. Hint: assume that all of the mass is in a point at the center and use what you know about the Kepler problem in the solar system (think about orbits). How reasonable is the assumption of constant velocity?

b. Under our assumptions, the density in the gain region can be written as

$$\rho(r) = \rho_0 \left(\frac{r_s}{r}\right)^2 . \qquad (1)$$

Justify the r-dependence in the above equation. Calculate the amount of mass in the gain region (in both grams and solar masses) for $\rho_0 = 10^9$ g cm^{-3} and $r_s = 150$ km. Calculate the approximate gravitational binding energy (in ergs) for the matter in the gain region.

c. Use equation (3.39) in the text to calculate the fraction of neutrinos absorbed as they stream radially outwards through the gain region. Assume the neutrino cross-section given in problem 4b) of problem set 1 and a neutrino energy of 20 MeV. Since the gain region is outside of the neutrinosphere, you should find a fraction much smaller than one.

d. The heating from the absorbed neutrinos of a particular parcel of matter will occur during the time that the matter takes to move through the gain region. Use this idea to make a simple estimate of the energy gained by an amount of matter equal to that in the gain region due to neutrino absorption as the matter falls through the region. Use a neutrino luminosity of 7×10^{52} erg s^{-1}. Compare this energy to the gravitational binding energy of this matter estimated in part b). Does the absorption of energy by neutrinos provide sufficient energy to unbind the matter in the gain region?

2. Assume that a newborn neutron star radiates 10^{53} erg in neutrinos. Show that an asymmetry in the emission as small as 1% (say, the proto-neutron star radiates slightly more neutrinos in the positive than in the negative z direction) is enough to impart a kick of several hundred km s^{-1} to the neutron star. Assume a mass of 1.4 M$_\odot$ and use the fact that the neutrinos involved are ultrarelativistic particles. All I am looking for in this problem is an order-of-magnitude calculation.

3. Scientists have placed an upper limit on the mass of the electron neutrino based on the 19 neutrinos detected from SN1987A. If the neutrino has a mass, neutrinos of different energies will have different speeds. An instantaneous pulse of neutrinos with a variety of energies will spread out as it travels the large distance between the LMC and Earth, with the more energetic

neutrinos arriving first. Show that the travel time of ultrarelativistic neutrinos of rest mass m_0 and energy E from a distance D to Earth is given approximately by

$$t \simeq (D/c)\left[1 + (1/2)(m_0 c^2/E)^2\right]. \tag{2}$$

Then show that the delay relative to a massless neutrino is

$$\Delta t \simeq (2.5\ \mathrm{s})\left(\frac{m_0 c^2}{10\ \mathrm{eV}}\right)^2\left(\frac{10\ \mathrm{MeV}}{E}\right)^2. \tag{3}$$

Use the data in the figure handed out in class giving neutrino energies and arrival times to provide an approximate upper limit on the neutrino mass. This is only an upper limit because the neutrinos actually take about 10 s to diffuse out the hot proto-neutron star.

4. In one model for a Type Ia supernova, a carbon white dwarf of 1.4 $M_\odot$ explosively burns to nuclear statistical equilibrium, producing “iron-group” elements (elements whose nuclear binding energy is similar to that of Fe), 0.7 $M_\odot$ of which is ^{56}Ni.

a. Show that the entire star can be disrupted, and estimate the average velocity of the ejecta. Assume that the radius of the white dwarf is 2×10^6 m and use a simple estimate for the initial gravitational binding energy of the white dwarf.

b. The ^{56}Ni produced by the supernova beta-decays to ^{56}Co, which then beta-decays to ^{56}Fe. These decays produce the energy emitted by the supernova in the form of electromagnetic radiation. Estimate the total amount of energy released by the radioactive decays. What is the ratio of this energy release to that released by the initial explosion?