Ph 442 Problem Set 1

Due: Thursday, September 17, 2009

1. This problem examines the transparency of our Galaxy to ultraviolet and X-ray photons with energies of i) 20 eV; ii) 200 eV; iii) 2 keV; and iv) 20 keV. Assume that the interstellar medium in our Galaxy contains an average density of 1 hydrogen atom per cubic centimeter.

   a. Use the analytic expression for the photoelectric absorption by the interstellar gas given in the Garmire notes to calculate the cross section per hydrogen atom at the four energies.

   b. Use the four cross section to calculate the mean-free-path for a photon in the interstellar medium. Express the mean-free-path in parsecs (1 pc $= 3.086 \times 10^{18}$ cm), the typical distance between stars in the plane of our Galaxy.

   c. Finally, calculate the hydrogen column density (atoms/cm^2) corresponding to the four mean-free-paths. These are the column densities that produce significant (reduction by 1/e) absorption at the four energies.

2. The intergalactic medium (the gas between galaxies) is ionized and has a mean electron density of about 2×10^{-7} cm^{-3}. Calculate the mean-free-path to Thompson scattering in the intergalactic medium. The cross section for Thompson scattering is $\sigma_T = (8\pi/3)r_0^2$, where $r_0 = e^2/(m_e c^2) = 2.82 \times 10^{-13}$ cm is the classical electron radius. (For those that prefer mks units, replacing e^2 in the expression for σ_T by $e^2/(4\pi\epsilon_0)$ converts it to mks.) Express your result in megaparsecs. The size of the visible universe is about the speed of light times the time since the Big Bang, 13.6 Gyrs, or 4300 Mpc. Can we expect to see x-ray sources at "cosmologically interesting" distances (i.e., at a significant fraction of the size of the visible universe)? Your calculation above ignored the changing density of the electrons due to the expansion of the universe. By what factor would the density have to increase to reduce the mean-free-path to 4300 Mpc?

3. Assume that 0.075 M$_\odot$ of ^{56}Co was produced by the decay of ^{56}Ni following the explosion of SN1987A.

   a. Estimate the amount of energy released per second through the radioactive decay of ^{56}Co just after its formation and 1 yr after the explosion. Express your answers in both ergs s^{-1} and solar luminosities.

   b. All of the ^{56}Co decays produce a gamma-ray with an energy of of 847 keV and these were observed coming from SN1987A 0.5 yr after the explosion. (A gamma-ray with an energy of 1238 keV is also produced in 68% of the decays.) Estimate the rate of gamma-ray production at that time and use this to calculate the maximum flux of 847 keV gamma-rays that could be observed at Earth. Assume a distance to SN1987A of 50 kpc and express your flux in units of photons cm^{-2} s^{-1}.

   c. The observed flux in the 847 keV line at the above time was about 1.0×10^{-3} photons cm^{-2} s^{-1}. Describe the most likely reason why your result from part b) is higher than the observed flux.

4. This problem considers the neutrino burst generated by SN1987A in the Large Magellanic Cloud (LMC) at a distance of 50 kpc.

a. If 10^{53} erg were released in neutrinos of mean energy 10 MeV each, how many neutrinos passed through a square centimeter on Earth?

b. For neutrino energies much less than 1 GeV, the cross section for scattering off of a nucleon is

$$\sigma_\nu = \sigma_0 \left(\frac{E_\nu}{m_e c^2} \right)^2, \tag{1}$$

where

$$\sigma_0 = \frac{4 G_F^2 m_e^2}{\hbar^4} = 1.76 \times 10^{-44} \text{ cm}^2 \tag{2}$$

and G_F is Fermi's constant. Show that the mean free path for neutrino scattering in matter with density ρ is

$$\ell_\nu = (2 \times 10^5 \text{ cm}) \left(\frac{E_\nu}{10 \text{ MeV}} \right)^{-2} \left(\frac{\rho}{10^{12} \text{ g cm}^{-3}} \right)^{-1}. \tag{3}$$

c. If the neutrinos perform a random walk out of the proto-neutron star at the center of the supernova exposion, the time to diffuse a radial distance, R, is given by

$$t_{diff} = \frac{R^2}{\ell_\nu c}. \tag{4}$$

Show that for a uniform density sphere of mass 1.4 $M_\odot$ the time for a neutrino to escape is

$$t_{diff} = (1.0 \times 10^{-2} \text{ s}) \left(\frac{\rho}{10^{12} \text{ g cm}^{-3}} \right)^{1/3} \left(\frac{E_\nu}{10 \text{ MeV}} \right)^2. \tag{5}$$

Evaluate this time for a typical (proto-)neutron star density of 1.0×10^{15} g cm^{-3} and a neutrino energy of 20 MeV.