Ph 441/541 Problem Set 4

Due: Friday, February 24, 2012

1. Electron and Radiation Equations of State:

   a. Consider a classical electron gas with number density $n_e = 10^{30}$ m^{-3} in equilibrium with radiation at temperature T. Find the value of the temperature, T_{eq}, at which the radiation pressure equals the electron pressure.

   b. Consider an ideal gas of degenerate, non-relativistic electrons with a concentration (i.e., number density) n and obtain an expression for the Fermi *energy*. Assume now that the gas has a temperature T such that the quantum concentration n_Q, given by Phillips Eq. (2.22), is equal to the actual concentration; quantum effects will be important in such a gas, but the electrons will not be completely degenerate. Find the ratio of kT to the Fermi energy.

2. Degenerate electrons:
Compare the relative importance of the electrostatic interactions between degenerate electrons and ions in a normal metal with a density of about 10^4 kg m^{-3} and in a white dwarf with a density of about 10^8 kg m^{-3}. Assume a composition of iron for the metal and of carbon for the white dwarf. In both cases, estimate the temperature below which the electrons are indeed degenerate.

3. Completely Degenerate Ideal Gas: the General Case (Ph 541 students only):
The pressure in an ideal degenerate electron gas is given by Phillips Eq. (2.31) if the electrons are non-relativistic and by Eq. (2.34) if the electrons are predominantly ultra-relativistic. Use the relativistic relation between energy and momentum, $\epsilon_p^2 = p^2c^2 + m^2c^4$, and show that the general expression for the pressure in an ideal degenerate gas is

$$P = K_{UR} n^{4/3} I(x),$$

where $x = p_F/mc$ and

$$I(x) = \frac{3}{2x^4}\left[x(1+x^2)^{1/2}\left(\frac{2x^2}{3} - 1\right) + \ln\left[x + (1+x^2)^{1/2}\right]\right].$$

Confirm that, in the appropriate limits, this expression for the pressure reduces to Eqs. (2.31) and (2.34), respectively. (We will use this general expression for the pressure in an ideal degenerate gas when discussion white dwarfs.)