Ph 441/541 Problem Set 3

Due: Friday, February 17, 2012

1. Understanding the Main Sequence:

Simple physical arguments can yield an approximate understanding of the relations between mass ($\mathcal{M}$), radius ($\mathcal{R}$), luminosity ($\mathcal{L}$), and effective temperature (T_{eff}) of main sequence stars. Simple approximate relations between these quantities exist because main sequence stars share a similar structure, though this similarity starts to break down for the most massive and least massive stars.

a. Derive Phillips eq. 1.36,

$$L \approx \frac{(4\pi)^2}{3^5}\frac{\sigma}{k^4}G^4\overline{m}^4\langle\rho\rangle\ell\mathcal{M}^3, \tag{1}$$

from the definition of T_{eff}, $\mathcal{L} = 4\pi\mathcal{R}^2T_{eff}^4$ (eq. 1.32), the virial theorem estimate of the central temperature (eq. 1.31),

$$kT_I \approx \frac{G\mathcal{M}\overline{m}}{3\mathcal{R}}, \tag{2}$$

and the relation between T_I and T_{eff} given by radiative diffusion (eq. 1.35),

$$T_{eff} \approx \left(\frac{\ell}{\mathcal{R}}\right)^{1/4} T_I \tag{3}$$

Here, as in Phillips, $\langle\rho\rangle$ is the average density of the star, ℓ is the average photon mean-free-path in the star, and $\overline{m}$ is the average particle mass in the star.

b. Show that $\langle\rho\rangle\ell \approx \overline{m}/\overline{\sigma_{ph}}$, where $\overline{\sigma_{ph}}$ is the average photon absorption and/or scattering cross-section per particle in the star (not to be confused with the Stefan-Boltzmann constant, σ). For stars with both a similar composition, which is mostly true for stars near the Sun, and a similar T_I (see below), both $\overline{m}$ and $\overline{\sigma_{ph}}$ are approximately constant. Thus, eq. 1 implies $\mathcal{L} \propto \mathcal{M}^3$. This agrees reasonably well with the observed $\mathcal{L} \propto \mathcal{M}^{3.6}$ on the upper main sequence. The agreement is less good for stars less massive than the Sun, where convective energy transport becomes increasingly important.

c. We will see that the rate of energy generation by nuclear fusion in the centers of stars increases very rapidly with increasing temperature. Thus, the thermonuclear thermostat discussed on p. 24 of Phillips tends to produce nearly the same T_I for all main sequence stars. Assume that the energy generation rate per unit mass, $\mathcal{L}/\mathcal{M}$, is proportional to $\langle\rho\rangle T^\alpha$. Use this with the $\mathcal{M}$-$\mathcal{L}$ relation from part b) and the virial theorem to predict that

$$\mathcal{R} \propto \mathcal{M}^{(\alpha-1)/(\alpha+3)}\overline{m}^{(\alpha-5)/(\alpha+3)}\overline{\sigma_{ph}}^{1/(\alpha+3)} \tag{4}$$

for main sequence stars. Evaluate the exponents for $\alpha = 17$ (energy generation by the CNO cycle — dominant for stars more massive than the Sun) and for $\alpha = 4$ (energy generation by the p-p chain — dominant for stars less massive than the Sun). The agreement with the observed $\mathcal{M}$-$\mathcal{R}$ relation for the upper main sequence is good. The agreement is again poorer for the lower main sequence , where $\mathcal{R} \propto \mathcal{M}^{0.9}$ is observed.

d. The initial composition of a star can also affect the main sequence. Stars differ primarily in the abundance of the so-called "metals", elements heavier than helium. These play an important role because they provide most of the opacity in the interiors of stars — either through bound-free absorption by their inner electrons or, when completely ionized, by producing numerous electrons, which Thompson-scatter photons. However, we will begin the examination of the role of composition by showing that it does not have a big effect on the average mass per particle, $\overline{m}$. Stellar composition is traditionally specified by X, Y, and Z, which are the *mass* fractions of hydrogen, helium, and metals, respectively. Assume that the metals have a mass per particle of $2m_A$ when completely ionized, where m_A is the atomic mass unit (the ratio is $12m_A/7$ for ^{12}C and $16m_A/9$ for ^{16}O, for example). Show that a completely ionized gas has

$$\overline{m} = \frac{m_A}{2X + 3Y/4 + Z/2}. \tag{5}$$

[Hint: it is useful to realize that $\overline{m} = \rho/n$, where n is the number density of the gas.] Evaluate $\overline{m}$ for both the solar composition of $X = 0.715$, $Y = 0.271$, and $Z = 0.014$ and a primordial metal-free composition of $X = 0.752$ and $Y = 0.248$. The two values of $\overline{m}$ differ only slightly.

e. For a fixed $\mathcal{M}$, does $\mathcal{L}$ increase or decrease as Z decreases? Similarly, does $\mathcal{R}$ increase or decrease? Without specifying the details of the opacity, we cannot say how rapid the increase or decrease is.

f. (Ph 541 students only) For a fixed $\mathcal{M}$, does T_{eff} increase or decrease as Z decreases?

2. Extrasolar Planets:

The same tools that are used to measure the masses of stars in binary systems are used to study extrasolar planets. In this case, the radial velocity measurements of the star show its motion around the star-planet center of mass. A recent study found that the star HD 85512 shows a sinusoidal variation in radial velocity with an amplitude of 0.769 ± 0.090 m s^{-1} (*i.e.*, a full range of twice that value; and, no, the unit of m s^{-1} is not a typo) and a period of 58.43 ± 0.13 days.

a. Look up HD 85512 in Simbad and report its spectral type, V-band magnitude, $B-V$ color, and parallax. Use the parallax to calculate its distance.

b. Use the table from Gray handed out in class to find the expected mass, radius, and M_V of the star. Also calculate M_V from V and the distance (assume negligible extinction). Comment on possible reasons for any difference in the two M_V values.

c. Calculate the orbital separation between the star and the planet in astronomical units. What is the maximum possible angular separation, in arcseconds, as seen from Earth?

d. Assuming that the orbit is viewed edge-on, what is the ratio of the mass of the planet to that of the star? What is the mass of the planet in units of Earth masses?

3. Perspective Gradient (Ph 541 students only):

Consider a gravitationally bound star cluster or galaxy moving through space with a radial velocity, v_r, and tangential velocity, v_t, with respect to the Sun. Assume that the observed center of the object is its center of mass. Thus, stars near the center will have an average radial velocity of v_r. Away from the center, the average velocity will differ because of our changed perspective in addition to any change due to internal motions.

a. Calculate how the changing perspective will cause the observed average radial velocity of the galaxy or cluster to vary with angular distance, θ, from the center along the direction of the tangential velocity.

b. The Fornax dwarf satellite galaxy of our Milky Way Galaxy has an angular diameter of about 60 arcminutes, $v_r = 55.2$ km s^{-1}, and $v_t = 82$ km s^{-1}. Evaluate the size of this perspective effect on v_r at the edge of the galaxy.

c. Briefly describe how a measurement of the perspective gradient in v_r and the proper motion of the center of a system could be used to determine the distance to the object.