Measuring the Wavelength of Light with a Steel Ruler

PURPOSE

The purpose of this lab is to measure the wavelength of light from a He-Ne Laser using a steel ruler and a meter stick.

HOMEWORK:

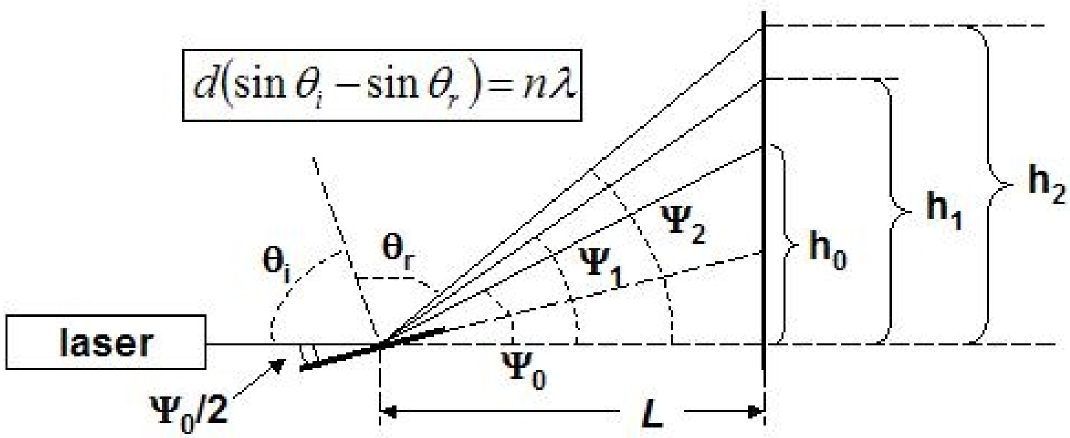
Taylor Problems 3.25, 3.30, 3.31.
Problem: A diffraction grating with d=1 micron is illuminated at normal incidence. The first diffraction peak is at an angle of 20 degrees with respect to the grating normal. What is the wavelength of the light?
Recommended reading: Any introduction to diffraction and interference of light (wave optics). For example, E. Hecht, Optics, Chapter 10; D. Giancoli, Physics, Chapter 24; F. Jenkins and H. White, Fundamentals of Optics, Chapters 15 and 17.

INTRODUCTION

When light waves from two ***coherent*** sources cross, they interfere with each other and the intensity of light is redistributed in space. As a result, on a screen illuminated by two coherent sources of light, one can observe an alternating pattern of bright and dark regions. If the wave peaks are in phase, there will be the so-called constructive interference and the intensity will be a maximum, while if they are of the same amplitude, but exactly out of phase, the intensity will be zero. The interference pattern can be generated in a number of ways; in this particular experiment, a *diffraction grating* will be used for this purpose. A diffraction grating consists of a series of regularly spaced scratches or absorbing lines which serve to remove parts of the incident wave front. Light wave reflected from the grating can be considered as a superposition of waves emitted by coherent secondary sources (recall what Huygens principle is about). The interference pattern observed on the screen is due to two effects: (a) diffraction of light waves on a single scratch and (b) interference between the light waves emitted by different secondary sources of light. The *maxima* in the interference pattern, produced by the constructive interference of the waves that arrive at the screen in phase, are given by

$$d(\sin\theta_i - \sin\theta_r) = n\lambda \quad , \qquad (1)$$

where *d* is the spacing between grating lines, λ is the wavelength of the light, *n* is the order of the diffraction maxima, θ_i is the angle of incidence, and θ_n is the angle of diffraction of the n^{th} order maximum, measured from the normal. In the experiment, you will measure the distance between the diffraction maxima on the screen, $h_{i+1} - h_i$ (see the Figure). Let's rewrite the condition (1) for constructive interference in terms of h_i:



$$\theta_i = 90^0 - \frac{\Psi_0}{2} \qquad \theta_r = 90^0 - \Psi_n + \frac{\Psi_0}{2} \qquad \tan\Psi_0 = \frac{h_0}{L} \qquad \tan\Psi_n = \frac{h_n}{L}$$

$$n\lambda = d\left[\sin\left(90^0 - \frac{\Psi_0}{2}\right) - \sin\left(90^0 - \Psi_n + \frac{\Psi_0}{2}\right)\right] =$$

$$d\left[\cos\left(\frac{\Psi_0}{2}\right) - \cos\left(\Psi_n - \frac{\Psi_0}{2}\right)\right] = \left\{\cos x \approx 1 - \frac{x^2}{2}\right\} =$$

$$\frac{d}{2}\left[\left(\Psi_n - \frac{\Psi_0}{2}\right)^2 - \left(\frac{\Psi_0}{2}\right)^2\right] = \{\tan x \approx x\} =$$

$$\frac{d}{2}\left[\left(\frac{h_n}{L} - \frac{h_0}{2L}\right)^2 - \left(\frac{h_0}{2L}\right)^2\right] = \frac{d}{2L^2}\left[(h_n)^2 - h_n h_0\right]$$

Note that ***h*$_0$** and ***h*$_n$** are two ***independent*** parameters. Consider two limiting cases:

$h_0 << h_n$ ($\Psi_0 << \Psi_n$)

$$n\lambda = \frac{d}{2L^2}\left[(h_n)^2 - h_n h_0\right] \approx \frac{d}{2L^2}(h_n)^2 \Rightarrow \quad h_n \propto \sqrt{n}$$

$h_0 \sim h_n$ ($\Psi_0 \sim \Psi_n$)

$$n\lambda = \frac{d}{2L^2}\left[(h_n)^2 - h_n h_0\right] \approx \frac{d}{2L^2} h_0 (h_n - h_0) \Rightarrow \quad h_n - h_0 \propto n$$

This derivation explains why the distance between adjacent maxima decreases with *n* in the former case. In the later case, the maxima are almost equidistant.

PROCEDURE

Safety reminder: Laser beams can be dangerous to eyes so do not look into a laser beam and do not point a laser near other people.

Use the ruled face of the steel ruler as a diffraction grating at glancing incidence. Before starting the measurement, estimate how precisely you must measure the angles to determine the wavelength of the laser light to about 2%. The wavelength is 632.8 nm. You will set up the laser so that the light is nearly parallel to the ruler. Recall that θ_n and θ_i are measured relative to a line perpendicular to the ruler, which means they will be near 90^o. Make a rough estimate of how far the diffraction pattern must be from the ruler in order for you to obtain 2% uncertainty.

Make a detailed drawing of your experimental setup, showing the light paths, the relevant angles, and give a derivation of Eq. (1). Which two light sources can be considered as "coherent"? Explain.

Set up the laser beam at a small angle with respect to the horizontal direction (the laser points slightly down). Use a piece of paper as the screen, and mark the position of the beam (this is the reference point for further measurements). Position the jack with the ruler close to the laser, slowly lift up the ruler using the jack, insert the ruler into the beam, and observe the diffraction pattern on the screen. Mark the positions of diffraction maxima on the paper for later analysis. Make sure all the maxima are labeled such that you can identify the order of diffraction later. Make the necessary measurements needed to determine the wavelength. Use several maxima and average the values to get the final wavelength. Try both the 1/64 and 1/32 inch scales of the ruler. Do they agree? Which should give a more precise result?

If the wavelengths derived from the different diffraction spots disagree significantly with each other, it is an indication that something is wrong. Most likely, you have not assigned the diffraction order *m* correctly to the various peaks. Check the measurements and the analysis until you have fixed the problem.