PROPAGATION OF ERROR

HOMEWORK

Required Reading: John R. Taylor, "An Introduction to Error Analysis", Chapters 1-3.
Do Problems 2.19, 2.27, 3.7, 3.24, 3.34, 3.46
Optional Reading: Reginald J. Stephenson, "Mechanics and Properties of Matter", 3rd Edition (QA 807.S83 1969), Sections 8.9,8.10. Also, Handbook of Physics, 2nd edition, (QC 21.C7 1967) p3-76.

EXPERIMENT

Purpose: To measure the modulus of rigidity of steel. To understand how the measurement uncertainties in one or more directly measured quantities propagate through a calculation to affect the uncertainty on the calculated quantity.

BACKGROUND – Propagation of Uncertainties

The determination of a physical quantity can only rarely be made by a single direct measurement. Usually several different quantities must be measured and those results combined to calculate the quantity of interest. The values obtained in each measurement always have an associated uncertainty which can be determined by either estimating the precision with which the measurement can be made or by repeating the measurement a number of times and calculating the standard deviation. The calculated quantity of interest will have an uncertainty due to the uncertainties in the contributing measured quantities. A general formula for calculating this uncertainty can be derived for the usual case where the uncertainties in the measured quantities are independent and random (see Taylor pg. 73). We state this result below.

Let x, ..., z be the measured quantities with uncertainties δz, δy, δz. Then the uncertainty in the quantity of interest q(x, ..., z) is

(1) $$\delta q = \sqrt{\left(\frac{\partial q}{\partial x}\delta x\right)^2 + \left(\frac{\partial q}{\partial y}\delta y\right)^2 + \left(\frac{\partial q}{\partial z}\delta z\right)^2}$$

The uncertainty on each independent quantity can be estimated by measuring that quantity several times. The best estimate of the standard deviation of the distribution of x-measurements, σ_x, is then (Taylor pg.100)

(2) $$\sigma_x = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(x_i - \bar{x})^2}$$

where N is the number of measurements. This corresponds to the uncertainty of any single measurement of x. However, when you have several measurements of x, it is usual to use the mean of x in the calculation of q, and then the uncertainty δx is reduced (divided by the square root of N), i.e., the uncertainty of the mean of N measurements is $\delta x = \sigma_x N^{-1/2}$.

BACKGROUND – Torsion Pendulum

In this experiment you will determine the value of the modulus of rigidity *M* of a steel wire, estimate the error in your determination, and compare your result with the handbook value. To determine *M* you will measure the period of a torsional pendulum consisting of a rectangular brass rod suspended from the wire. When the rod is turned horizontally through an angle θ and released it will experience a restoring torque due to the twisted support wire. This torque is $\tau = -k\theta$, where k is the torque constant of the wire (torque per unit angle of twist). Then by Newton's 2nd law

(3) $$-k\,\theta = I\frac{d^2\theta}{dt^2}$$

where I is the moment of inertia. We see that the torsional motion is simple harmonic with an angular frequency ω and period T of

(4) $$\omega = \sqrt{\frac{k}{I}} \qquad T = 2\pi\sqrt{\frac{I}{k}}$$

The moment of inertia for a rectangular rod suspended at the center of mass is calculated from its measured mass *m*, width *a* and length *b*:

(5) $$I = \frac{m}{12}\left(a^2 + b^2\right)$$

The torsional constant *k* is related to the modulus of rigidity. For a wire of diameter *d* and length *L*

(6) $$k = \frac{\pi d^4 M}{32L}$$

We can eliminate *k* from Eq.(6) using Eq.(4) and solve for *M*

(7) $$M = 128\,\pi\frac{L\,I}{d^4T^2} = \frac{32\pi}{3}\frac{Lm(a^2+b^2)}{d^4T^2}$$

Thus by measuring six quantities -- the period *T*, the length *L* and diameter *d* of the wire, the mass *m*, the width *a* and length *b* of the rod -- we can calculate the modulus of rigidity. The uncertainty on the value of *M* will be a combination of the uncertainties in these six measurements.

PROCEDURE

1. The goal is to determine the modulus of rigidity of steel. First, measure the six quantities quickly, taking reasonable but not extreme care with the measurements. (Be sure to measure a full period, not half period.) Calculate the moment of inertia I first, then the modulus M using I. Pay careful attention to proper units, it is easy to make mistakes. (Try mks units. Some reference books use the unit of dynes/cm^2 where the dyne = 10^{-5} Newton.) Roughly estimate the errors in each measurement and the uncertainty in M.

2. From your results in step 1 decide which measurements are most crucial in determining the final uncertainty in M and which ones make only a minor contribution. You may decide that some measurements are accurate enough already and do not need improvement. Others may be crucial and need to be made extremely carefully.

3. Examine your measuring techniques for possible systematic errors (see Taylor pg.11,94). Alter your techniques or carry out necessary calibrations etc. to minimize these errors. Recall that the period for a harmonic oscillator is independent of amplitude, so there is no need to induce large oscillations of the block (and in fact these can make cause a non-linearity in the restoring force which will make the measurement less accurate). You can reduce the error by timing several oscillations and dividing the time by the total number of oscillations.

4. Now repeat the experiment carefully, making multiple measurements of important quantities and calculate your final value of M. Carefully estimate the uncertainty in your value. If every measured value of a some quantity is the same, take σ to be half the least significant figure. Set up a Matlab M-file to record the data, make the required calculations, and determine the final result and its uncertainty. Turn in the printout of your Matlab M-file with your lab report.

5. Compare your value for M with the handbook value. There are several books in the reference section of the Physics Library on the third floor of Serin that give the handbook value for steel. Since *M* varies with the carbon content, these books will give a range of values rather than a specific number. The values for *M* will be found in tables of Elastic Moduli. The modulus of elasticity is also known as the modulus of torsion or the shear modulus of elasticity.

6. Your lab report will include the M-file with your data and the calculation of *I* and *M* and their errors, a short description of the measurement technique and instruments, and a

discussion of the sources and propagations of error and suggestions for ways to improve the accuracy and precision of the experiment. Be specific and avoid generalities that could be applied to any experiment.