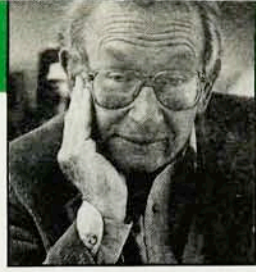


SPIN GLASS V: REAL POWER BROUGHT TO BEAR

Philip W. Anderson

Gérard Toulouse had always been interested in the spin glass problem. In 1977, subsequent to the work I discussed in my last column (September 1988, page 9), Gérard, then at the Ecole Normale Supérieure in Paris, began to discuss the problem with the powerful "Cargèse" group of theoretical physicists in Paris and Rome: Cyrano de Dominicis, Giorgio Parisi and Miguel Virasoro in particular, and later Bernard Derrida, Nick Sourlas and others. Gérard was the originator of the formal theory of "frustration" (I believe I introduced the *term* originally) as *the* important feature of the spin glass problem. Because the exchange bonds J between spins have random signs in most circuits (loops) of spins returning upon themselves, not all the spins can be made "happy"—hence the "frustration." In a square of four spins, for instance, only if an even number of the J's have the same sign can one satisfy everybody—that is, find a unique minimum-energy configuration of the four spins. Since the world is made up of systems of conflicting desires, from game strategies to a group of people choosing a menu, one begins to see that the spin glass is not that bad a model for many aspects of life.

It was Giorgio Parisi who developed the replica-symmetry-breaking scheme that solved the problems, such as negative entropy, that had been troubling us. You will remember from my column of June 1988 (page 9) that Sam Edwards had introduced the "replica" scheme of making m identical copies $1, \ldots, \alpha, \ldots, m$ of the same system and calculating the average of the product of all m partition functions. One then schematically takes the limit $m \to 0$ and uses the formula

$$\lim_{m \to 0} \frac{Z^m - 1}{m} = \ln Z$$

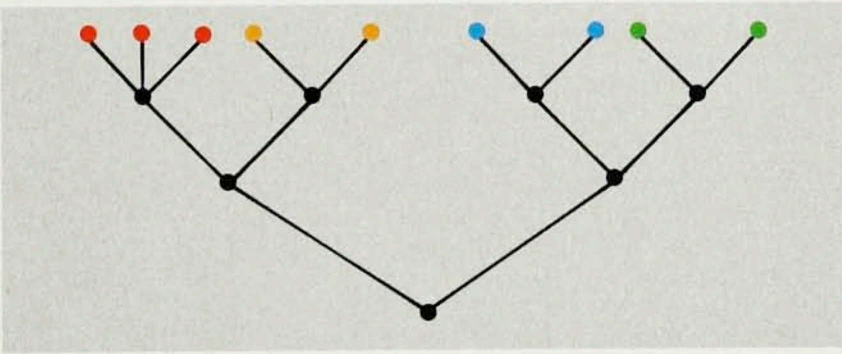
to calculate the free energy

$$F = -kT \langle \ln Z \rangle_{\text{ave}}$$

David Thouless and Jairo de Almeida later showed that not all pairs of copies gave the same average correlation

$$q_{\alpha\beta} = \langle S_i^\alpha S_i^\beta \rangle_{\text{ave over } i}$$

This finding was called "replica symmetry breaking." Giorgio was able to produce a form of $q_{\alpha\beta}$ that worked, in the sense that it was a self-consistent and stable solution of the equations in this replica formalism. It would not be wise for me to go through the complicated mathematical structure that evolved from this beautiful and unexpected solution. Instead I shall try to describe what was eventually understood about its implications—with contributions from Toulouse, Virasoro, and later Haim Sompolinsky, Peter Young, Richard Palmer and many others.



Ultrametric tree. This structure is a convenient way to represent the degree of resemblance between spin glass states (colored dots). The overlap between any pair of states of the same color is q_1; that between "red" and "orange" or between "blue" and "green" states is $q_2 < q_1$. Similarly, the overlap of any of the five states on the left with any of the four states on the right is $q_3 < q_2$. Thus the overlap between two states depends on how deep into the tree one has to go to find a node (black dot) that connects them. One may verify from the figure the amazing property that when any three states are picked at random at least two of the overlaps are equal.

What Giorgio's solution means is that at any temperature below T_c there is no *unique* locally stable thermodynamic state that solves the "TAP" mean-field equations I described in my last column, but rather many such states, which resemble one another to different degrees. Each replica α corresponds to a different solution of the TAP equations; the solutions can be thought of as clusters of states in the N-dimensional configuration space of the N spins. The TAP equations are obtained when the thermodynamic average is restricted to these local clusters of states. The off-diagonal terms in the order parameter $q_{\alpha\beta}$, which represent the average overlap between states in the cluster belonging to replica (solution) α and those belonging to replica β, are a measure of the degree of resemblance between clusters α and β. The diagonal elements $q_{\alpha\alpha}$, which are all the same, are the average overlaps of

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states within a given replica, or cluster. There is a hierarchy of overlaps, with $q_{\alpha\alpha} \equiv q_0$ being the largest. The next in value is q_1, the overlap between the inequivalent groups of solutions α and β that are closest in the configuration space. Then there is a $q_2 < q_1$ and so on.

At T_c the solutions begin to separate, and the "distance" between them, measured by the deviation of their overlaps from unity, increases until, at $T = 0$, q_0 is 1 but the smallest $q_{\alpha\beta}$ may be nearly zero. Of course, the q's have a continuous distribution in a large ($N \to \infty$) system. Toulouse, Parisi and their collaborators showed that the distances, or overlaps, between different states implied by Parisi's form for the $q_{\alpha\beta}$ could be described by what is called an ultrametric tree. The figure on page 9 shows such a tree, in which no solution in the "red" group is any closer than q_2 to any in the "orange" group.

It is not at all surprising, then, that finding the "best" solution by computer simulations had been impossible: The solutions that separated at T_c became increasingly different as T was lowered. From thermodynamics and the extensive nature of the thermodynamic variables such as entropy and energy, one can show explicitly, as I did, that the only route from one set of solutions to another—through configuration space—passes over energy barriers whose height grows with N, the total number of spins. Thus if you try to get from one solution to another by flipping spins a few at a time, you must make flips that increase the energy by amounts of order N before you can ever get to one of the other solutions or, in particular, to the *best* one. Thus one can represent the solutions as deep valleys connected only by very high passes in a "rugged energy landscape" (to use Stu Kauffman's terms). This is a remarkable result—how truly remarkable and powerful we are only beginning to understand. It implies, among other things, a new thermodynamics—a thermodynamics of systems that are never in thermodynamic equilibrium. Richard Palmer and I called these systems "nonergodic." That one can nonetheless use statistical mechanical methods to get not only the quantitative solutions relevant to such systems but also the structure of the set of solutions is, to say the least, fantastic.

Next time I shall begin my discussion of the implications of this work in fields as far apart as computer science, biology and neuroscience, which normally have been quite outside the purview of physics. ■