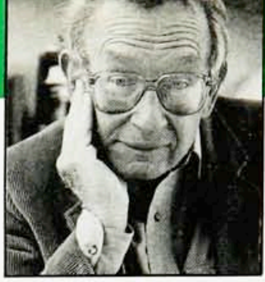


SPIN GLASS IV: GLIMMERINGS OF TROUBLE

Philip W. Anderson

In my last column (June, page 9), it seemed as though Sam Edwards's beautiful "replica" scheme had brought us to a highly satisfactory resolution of the old problem of magnetic systems with random exchange interactions—what we now call "spin glasses." (In the replica method, one calculates the partition function of n replicas α of the same random Hamiltonian, averages over the randomness and takes the logarithm by studying the formal limit as $n \to 0$: an indirect, shaky but often useful procedure.) In 1975 David Sherrington, who had been Sam's student and is now at Imperial College, London, tried applying the methods and ideas of the Edwards–Anderson paper to an especially simple model in which the "mean field" version should certainly be exact. In Sherrington's model, every spin in a macroscopic sample of N spins is connected by a random exchange integral $J_{ij} \times 1/\sqrt{N}$ to every other spin. This is precisely the kind of artificial system for which mean-field theory is exact in other magnetic models. Sherrington brought the model with him that summer on a visit to IBM (Yorktown Heights), where he worked with Scott Kirkpatrick. The model is now famous as the SK model. Their conclusion was that the EA method led to a solution that, while superficially plausible, was unequivocally *nonsense*—specifically, they showed that as the temperature approached zero the calculated entropy passed through zero and became negative. Since entropy is the log of an integer (the number of states at energy E), its acquiring a negative value is forbidden in statistical mechanics. The energy near $T = 0$ also seemed to be a little lower (by about 2 percent) in the SK solution than the best that Scott could achieve by simulating the model on a computer.

Naturally, everyone at first assumed that the replica method itself was at fault. In fact David Thouless, Richard Palmer and I set out to produce a solution directly, without the replica method. This so-called TAP theory (1977) adapted the ancient cavity-field method of Lars Onsager and Hans Bethe to include a local-field correction for the response of all the spins affected by the fluctuations of a given spin. This correction, which is absolutely negligible (of order $1/N$) in the corresponding long-range ferromagnet, is finite here, changing T_c by a factor of 2, for instance. But we could "prove" that all further corrections *were* negligible. The results agreed with the SK findings near and above T_c, where in fact we now know that both solutions are right, but they deviated subtly below T_c. One important difference was that we got rid of the negative entropy of the SK solution.

Again we thought we had the answer, and again we were to be disappointed, though the problem surfaced in more subtle ways this time. Central to the TAP solution is a mean-field equation,

$$m_i = \tanh \frac{h_i}{2k_B T}$$

where

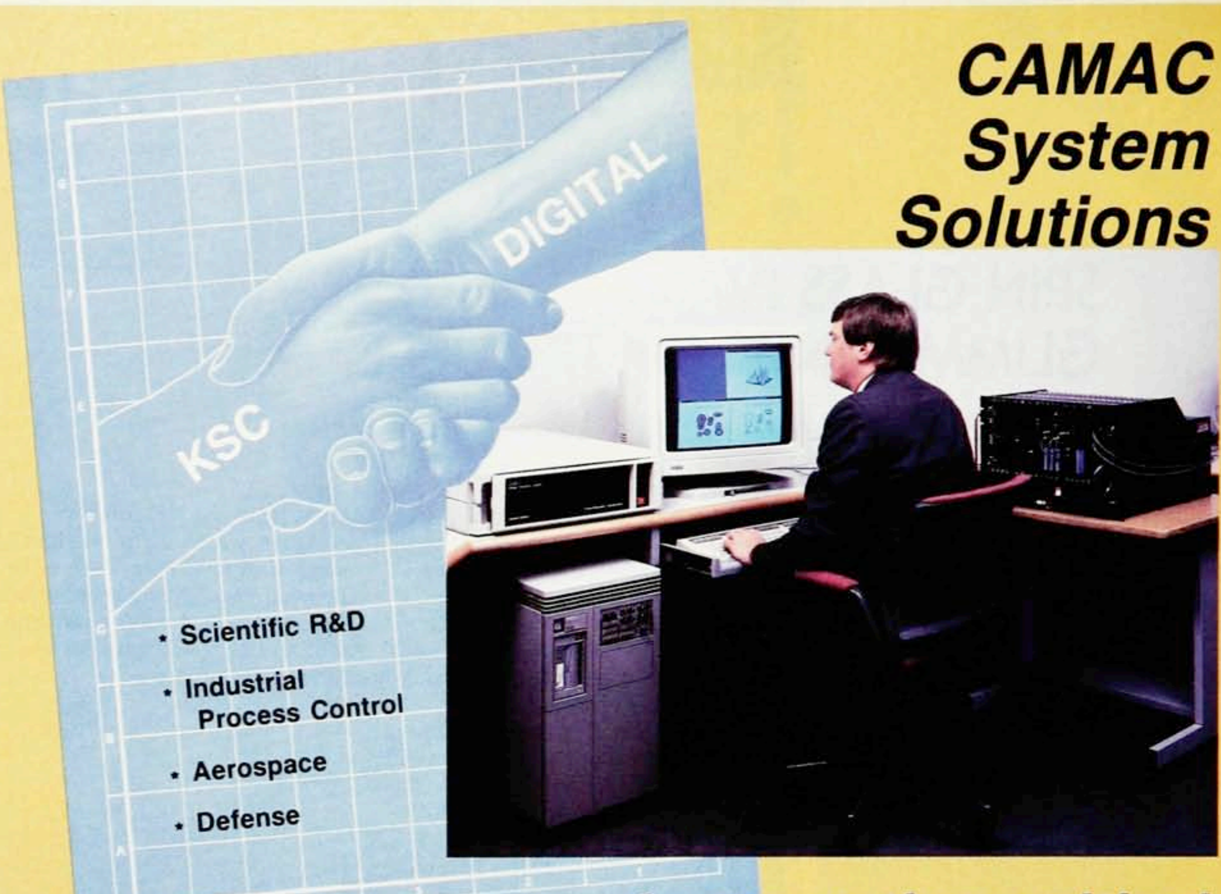
$$h_i = \sum_j J_{ij} m_j - (\text{local field correction})$$

Here m_i is the mean magnetization at site i. Unlike in the simple case of a ferromagnet, where similar equations are encountered, looking for a nontrivial solution for the magnetization m_i in this equation involves an infinite random-matrix problem at every T. Near T_c, this problem appeared to be just expressible in terms of the known statistical properties of the eigenvalues of the random matrix J, and near zero it depends only on properties of "the" solution at $T = 0$. The former case we solved in lowest order, and it seemed to look OK; and for the latter

"WHAT YOU HAVE DONE, GRUNDIG, IS HELP CONTRIBUTE TO A DIS-UNIFIED THEORY."

Philip Anderson is a condensed matter theorist whose work has also had impact on field theory, astrophysics, computer science and biology. He is Joseph Henry Professor of Physics at Princeton University.

CAMAC System Solutions



. . . for your time-critical data acquisition and control needs

As cooperative marketing partners, KSC and Digital Equipment Corporation provide high-performance system solutions based on the CAMAC standard (IEEE-583) for Computer Automated Measurement and Control. KSC systems offer you these unique benefits:

an established international standard. CAMAC is a *fully supported* IEEE, ANSI, and IEC standard.

full Digital compatibility. CAMAC interfaces to the *entire family* of LSI, PDP, MicroVAX, VAX, and VAXBI computers, including DRQ3B and DRB32 interfaces.

high data throughput. CAMAC systems transfer data at three million bytes per second over distances as great as three kilometers.

extensive I/O capability. You can choose your interface modules from over 150 different types, including 16-bit A/D and D/A converters, discrete input and output, counters, and 100 megasample per second transient recorders.

open architecture. With CAMAC, you can start as small as you like, expand when and how you want.

powerful distributed systems. The CAMAC serial highway supports distributed processing at as many as 62 remote stations.

supporting software. CAMAC software routines for VAXlab Software Library (VSL) are fully compatible with Digital's Realtime Calling Standards (RTCS). Menu-driven packages and software drivers are also available.

field proven. CAMAC is used worldwide in such applications as jet engine testing, furnace control, flight simulation, glass manufacturing, fusion research, and particle accelerator control.

Since 1970, we have been supplying state-of-the-art CAMAC systems for scientific, industrial, and aerospace/defense markets. To discuss solutions for your realtime data acquisition and control application needs, call us today.

KineticSystems Corporation

11 Maryknoll Drive, Lockport, Illinois 60441 • 815 838 0005 • TWX: 910 3612 • FAX: 815 838 4424
Switzerland: *Geneva:* (022) 98 44 45 • *Zuchwil:* (065) 25 29 25

digital™ Cooperative Marketing Program

case, Richard set out to calculate "typical" ground state solutions (that is, solutions at $T = 0$). The limiting form of the above set of random equations at $T = 0$ is

$$m_i = \mathrm{sign}(h_i)$$

$$h_i = \sum_j J_{ij} m_j$$

("Sign" just means that m points in the same direction as h and has the maximum possible magnitude.) Both Richard and Scott had been trying to solve these equations numerically for some time.

Both of them gradually came to the same paradoxical result: They could find no "the" solution to this set of equations. Instead, they found many, many solutions of nearly identical energy. They also noticed that it is very difficult, once one's computer has found one solution, to persuade it to move to another, even if the first has a much higher energy than the optimal one. Incidentally, both Richard and Scott found that the easiest way to find a new solution was to raise the temperature nearly to T_c and come back down again—a procedure that Scott called "simulated annealing."

This peculiar feature, enormously annoying at the time, was the beginning of one of the important discoveries of modern theoretical physics, a discovery comparable to that of chaos in its broad applicability to science. But we didn't quite understand that yet.

Because of this unusual feature, and also for other reasons—Thouless, for instance, was unhappy that our solution near T_c might not be quite stable—the TAP "solution" still did not satisfy. We also needed to know why the replica method had failed. Thouless and a student, Jairo de Almeida, soon discovered the rather unexpected reason. Below a certain line in magnetic field–temperature space, a solution with "replica symmetry"—that is, where every replica has the same correlation $q = q_{\alpha\beta}$ with every other replica—is dynamically unstable to "replica symmetry breaking." This implied that there was some new structure in $q_{\alpha\beta}$ that depended on α and β: According to the ideas that underlay the Edwards–Anderson paper, then, not every time you tried to compare one specimen of a system with another specimen of the *same* system would you get the same answer! Looking back, it seems obvious that this was closely related to the simulation problem, but it was a few years before we caught on to that. In my next column I'll try to explain the final resolution. ■