Phil Anderson And Gauge Symmetry Breaking

Edward Witten
Institute for Advanced Study
Princeton, NJ 08540

In this lecture, I am going to talk primarily about just one of the dozens of important papers that Phil Anderson has written. This paper was written just a little over 51 years ago, in 1962. I will set it in context by telling a little about the work that Julian Schwinger had done shortly before, and also we will recall Phil's earlier work on superconductivity that helped to set the stage. Then we will take a trip through later developments that occurred through the rest of the 1960's and beyond, and we will conclude with some observations about the present. (I won't talk about some early precursors such as Stueckelberg in 1938, but later we will get to the model introduced by Landau and Ginzburg in 1950.)

The title page of Phil's paper, which is called "Plasmons, Gauge Invariance, and Mass," and was received on November 8, 1962, can be found in fig. 1. As one can see, Phil starts out by citing the work of Julian Schwinger. The reference is to two very short papers that Schwinger had written, also published in 1962. To understand Phil's work, we should first take a look at Schwinger's contributions.

In the first paper (fig. 2), Schwinger argues somewhat abstractly that – in contrast to what we are familiar with in the case of electromagnetism – gauge invariance does not imply the existence of a massless spin one particle. A couple of things are worth noting here, apart from the fact that Schwinger was forward-thinking to even ask the question. One is that Schwinger was motivated by the strong interactions (there is no mention of weak interactions in the paper). The question he asks in the first paragraph is whether the conservation law of baryon number could be a gauge symmetry. There is an obvious problem, which is that we do not see a massless spin 1 particle coupled to baryon number. So Schwinger asked whether it is possible for baryon number (or something like baryon number) to be conserved because of a gauge symmetry, without the gauge symmetry producing a massless spin 1 particle.

The answer that Schwinger proposes – even in the first sentence of the abstract – is that gauge invariance does not necessarily imply the existence of a massless spin 1 particle if the coupling is large. His idea is that there is no massless particle if there is a pole in the current-current correlation function $\langle J_\mu(q)J_\nu(-q)\rangle$. The role of strong coupling is supposed to be to generate this pole. Thus, QED is weakly coupled, and has no such pole; Schwinger's point of view is that in a suitable gauge-invariant theory, strong coupling effects might produce such a pole.

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Plasmons, Gauge Invariance, and Mass

P. W. ANDERSON
Bell Telephone Laboratories, Murray Hill, New Jersey
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Schwinger has pointed out that the Yang-Mills vector boson implied by associating a generalized gauge transformation with a conservation law (of baryonic charge, for instance) does not necessarily have zero mass, if a certain criterion on the vacuum fluctuations of the generalized current is satisfied. We show that the theory of plasma oscillations is a simple nonrelativistic example exhibiting all of the features of Schwinger's idea. It is also shown that Schwinger's criterion that the vector field $m\neq 0$ implies that the matter spectrum before including the Yang-Mills interaction contains $m=0$, but that the example of superconductivity illustrates that the physical spectrum need not. Some comments on the relationship between these ideas and the zero-mass difficulty in theories with broken symmetries are given.

RECENTLY, Schwinger[1] has given an argument strongly suggesting that associating a gauge transformation with a local conservation law does not necessarily require the existence of a zero-mass vector boson. For instance, it had previously seemed impossible to describe the conservation of baryons in such a manner because of the absence of a zero-mass boson and of the accompanying long-range forces.[2] The problem of the mass of the bosons represents the major stumbling block in Sakurai's attempt to treat the dynamics of strongly interacting particles in terms of the Yang-Mills gauge fields which seem to be required to accompany the known conserved currents of baryon number and hypercharge.[3] (We use the term "Yang-Mills" in Sakurai's sense, to denote any generalized gauge field accompanying a local conservation law.)

The purpose of this article is to point out that the familiar plasmon theory of the free-electron gas exemplifies Schwinger's theory in a very straightforward manner. In the plasma, transverse electromagnetic waves do not propagate below the "plasma frequency," which is usually thought of as the frequency of long-wavelength longitudinal oscillation of the electron gas. At and above this frequency, three modes exist, in close analogy (except for problems of Galilean invariance implied by the inequivalent dispersion of longitudinal and transverse modes) with the massive vector boson mentioned by Schwinger. The plasma frequency is equivalent to the mass, while the finite density of electrons leading to divergent "vacuum" current fluctuations resembles the strong renormalized coupling of Schwinger's theory. In spite of the absence of low-frequency photons, gauge invariance and particle conservation are clearly satisfied in the plasma.

In fact, one can draw a direct parallel between the dielectric constant treatment of plasmon theory[4] and Schwinger's argument. Schwinger comments that the commutation relations for the gauge field A give us one sum rule for the vacuum fluctuations of A, while those for the matter field give a completely independent value for the fluctuations of matter current j. Since j is the source for A and the two are connected by field equations, the two sum rules are normally incompatible unless there is a contribution to the A rule from a free, homogeneous, weakly interacting, massless solution of the field equations. If, however, the source term is large enough, there can be no such contribution and the massless solutions cannot exist.

The usual theory of the plasmon does not treat the electromagnetic field quantum-mechanically or discuss vacuum fluctuations; yet there is a close relationship between the two arguments, and we, therefore, show that the quantum nature of the gauge field is irrelevant. Our argument is as follows:

The equation for the electromagnetic field is

$$p^2 A_\mu = (k^2-\omega^2)A_\mu(\mathbf{k},\omega) = 4\pi j_\mu(\mathbf{k},\omega).$$

[1] J. Schwinger, Phys. Rev. **125**, 397 (1962).
[2] T. D. Lee and C. N. Yang, Phys. Rev. **98**, 1501 (1955).
[3] J. J. Sakurai, Ann. Phys. (N. Y.) **11**, 1 (1961).
[4] P. Nozières and D. Pines, Phys. Rev. **109**, 741 (1958).

FIGURE 1. The first page of Phil Anderson's paper on gauge symmetry breaking, received November 8, 1962.

In Schwinger's second paper (fig. 3), he gives a concrete example of gauge invariance not implying the existence of a massless spin 1 particle. The example is based on a remarkable exact solution, not assuming weak coupling.

The model Schwinger solved was simply 1 + 1-dimensional Quantum Electrodynamics, with electrons of zero bare mass. The action is

$$I = -\frac{1}{4e^2}\int \mathrm{d}^2x\, F_{\mu\nu}F^{\mu\nu} + \int \mathrm{d}^2x\, \bar{\Psi} i \not{D} \Psi.$$

Nowadays this model – which is known as the Schwinger model – is usually solved simply and understandably (but surprisingly) by "bosonization," which converts it to a free theory (of the gauge field A_μ and a scalar field ϕ) with all the properties that Schwinger claimed. Schwinger's approach to solving it was more axiomatic.

The model is actually considered an important example that illustrates quite a few things, and not only what Schwinger had in mind. For instance, it is also used to illustrate the physics of a

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Gauge Invariance and Mass

JULIAN SCHWINGER
Harvard University, Cambridge, Massachusetts, and University of California, Los Angeles, California
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It is argued that the gauge invariance of a vector field does not necessarily imply zero mass for an associated particle if the current vector coupling is sufficiently strong. This situation may permit a deeper understanding of nucleonic charge conservation as a manifestation of a gauge invariance, without the obvious conflict with experience that a massless particle entails.

DOES the requirement of gauge invariance for a vector field coupled to a dynamical current imply the existence of a corresponding particle with zero mass? Although the answer to this question is invariably given in the affirmative,[1] the author has become convinced that there is no such necessary implication, once the assumption of weak coupling is removed. Thus the path to an understanding of nucleonic (baryonic) charge conservation as an aspect of a gauge invariance, in strict analogy with electric charge,[2] may be open for the first time.

One potential source of error should be recognized at the outset. A gauge-invariant system is not the continuous limit of one that fails to admit such an arbitrary Green's functions of other gauges have more complicated operator realizations, however, and will generally lack the positiveness properties of the radiation gauge.

Let us consider the simplest Green's function associated with the field $A_\mu(x)$, which can be derived from the unordered product

$$\langle A_\mu(x)A_\nu(x')\rangle = \int \frac{(dp)}{(2\pi)^3} e^{ip(x-x')} dm^2\, \eta_+(p)\delta(p^2+m^2)A_{\mu\nu}(p),$$

where the factor $\eta_+(p)\delta(p^2+m^2)$ enforces the spectral restriction to states with mass $m\geq 0$ and positive energy.

FIGURE 2. The first of Schwinger's two papers on gauge symmetry breaking.

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Gauge Invariance and Mass. II*

JULIAN SCHWINGER
Harvard University, Cambridge, Massachusetts
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The possibility that a vector gauge field can imply a nonzero mass particle is illustrated by the exact solution of a one-dimensional model.

IT has been remarked[1] that the gauge invariance of a vector field does not necessarily require the existence of a massless physical particle. In this note we shall add a few related comments and give a specific model for which an exact solution affirms this logical possibility. The model is the physical, if unworldly situation of electrodynamics in one spatial dimension, where the charge-bearing Dirac field has no associated mass constant. This example is rather unique since it is a simple model for which there is an exact divergence-free solution.[2]

and $\lambda^2>0$ unless $m=0$ is contained in the spectrum. Thus, it is necessary that λ vanish if $m=0$ is to appear as an isolated mass value in the physical spectrum. But it is also necessary that

$$s(0)=0,$$

such that

$$\int_{\to 0}^{\infty} \frac{dm^2}{m^2} s(m^2) < \infty,$$

for only then do we have a pole at $p^2=0$,

FIGURE 3. Schwinger's second paper on gauge symmetry breaking, in which he introduced and solved what is now called the Schwinger model.

gauge theory θ-angle, and after a small perturbation to give the electron a bare mass, it becomes a model of confinement of charged particles. However, although I was not in physics at the time, I suspect that Schwinger's extremely short paper mystified many of his contemporaries. His way of solving the model probably was a little abstract, and the whole thing probably seemed to revolve around peculiarities of $1+1$ dimensions.

Schwinger's concept was summarized in the last sentence of his first paper: "the essential point is embodied in the view that the observed physical world is the outcome of the dynamical play among underlying primary fields, and the relationship between these fundamental fields and the phenomenological particles can be comparatively remote, in contrast to the immediate correlation that is commonly assumed." In other words, in general, there need be no simple relationship between particles and fields – or in condensed matter physics, between bare electrons and nuclei and the emergent quasi-particles that give a more useful description at long distances.

Schwinger is saying that the situation that prevails in QED – in which the electron field corresponds to electrons, and the photon field corresponds to photons – results from the fact that this theory is weakly coupled. In a strongly coupled theory, there might be no simple correspondence between fields and particles. This was actually a very wise remark, probably putting Schwinger way ahead of his contemporaries in particle physics. And it is at the core of the way we now understand the strong interactions.

But Phil Anderson showed that Schwinger was actually not entirely correct about the specific question he was writing about – how to have gauge invariance without a massless spin one particle. To be more precise, everything that Schwinger said about strong coupling is true, but it is not the whole story. As Anderson showed, a weakly coupled vector meson might also acquire a mass, and here the essence of the matter is not strong coupling but symmetry breaking – the properties of the vacuum. In fact, Phil expressed a point of view that is quite opposite to Schwinger's, showing that not just strong coupling but even quantum mechanics is irrelevant to the problem of how to have gauge invariance without a massless spin one particle.

Phil's paper is largely devoted to two examples from well-established physics. He begins by saying that "the familiar plasmon theory of the free electron gas exemplifies Schwinger's theory in a very straightforward manner," with the plasma frequency, below which electromagnetic waves do not propagate, playing the role of the vector meson mass for Schwinger. He shows that the usual analysis of screening in a plasma can be put in close parallel with what Schwinger had said in the relativistic case. He also observes that the problem of screening in a plasma is usually understood classically, without invoking quantum mechanics, and deduces that "the quantum nature of the gauge field is irrelevant" to the question of how to have gauge invariance without a massless vector particle.

The second part of Phil's 1962 paper deals with an example that is even more incisive – superconductivity. The background to this was provided by a series of three papers that Phil had written in 1958 (shown in fig. 4 is the title page of the first of the three papers – which incidentally is the one he referred to in 1962). In these papers, Phil had analyzed gauge invariance and the fate of the "Goldstone" boson (the term is ahistorical as Goldstone had not yet formulated his relativistic theorem) in the BCS theory of superconductivity, showing that this mode combines with ordinary photons to become a gapped state of spin 1. Thus the electronic state of an ordinary BCS superconductor is truly gapless.

In the 1962 paper, Phil explains cogently how superconductivity illustrates the phenomenon described by Schwinger, in a context in which the gauge field is weakly coupled and the physics is well understood. Much of this paper reads just like what one would explain to a student today.

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Coherent Excited States in the Theory of Superconductivity: Gauge Invariance and the Meissner Effect

P. W. ANDERSON
Bell Telephone Laboratories, Murray Hill, New Jersey
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We discuss the coherent states generated in the Bardeen, Cooper, and Schrieffer theory of superconductivity by the momentum displacement operator $\rho_{\mathbf{Q}} = \sum_n \exp(i\mathbf{Q}\cdot\mathbf{r}_n)$. Without taking into account plasma effects, these states are like bound Cooper pairs with momentum $\hbar\mathbf{Q}$ and energies lying in the gap, and they play a central role in the explanation of the gauge invariance of the Meissner effect. Long-range Coulomb forces recombine them into plasmons with equations of motion unaffected by the gap. Central to the argument is the proof that the non-gauge-invariant terms in the Hamiltonian of Bardeen, Cooper, and Schrieffer have an effect on these states which vanishes in the weak-coupling limit.

I. INTRODUCTION

BUCKINGHAM[1] has questioned whether an energy-gap model of superconductivity, such as that of Bardeen, Cooper, and Schrieffer,[2] can explain the Meissner effect without violating a certain identity derived by Schafroth[3] on the basis of gauge invariance, and by Buckingham using essentially an f-sum rule. This identity is what causes the insulator, which also has an energy gap, to fail to show a Meissner effect; thus, Buckingham and Schafroth[4] argue, a proof of gauge invariance lies at the core of the problem of superconductivity, especially since the Hamiltonian used in B.C.S. is not gauge-invariant.

instead of zero. The total operator applied to Ψ_g leads to a linear combination of such states, which can be thought of as equivalent to a Cooper bound state[6] of a pair of electrons with nonzero momentum, superimposed on the B.C.S. ground state.

Our discussion of these problems is based on the following physical picture: any transverse excitation involves breaking up the phase coherence over the whole Fermi surface of at least one pair in the superconducting ground state, and so involves a loss of attractive binding energy. This causes the Meissner effect. Longitudinal excitations, however, such as those generated by $\rho_{\mathbf{Q}}$, do not break up phase coherence in the superconducting

FIGURE 4. The first of Anderson's three papers of 1958 on gauge invariance in the BCS model of superconductivity.

For example: "the way is now open for a degenerate vacuum theory of the Nambu type without any difficulties involving either zero-mass Yang-Mills gauge bosons or zero-mass Goldstone bosons. These two types of bosons seem capable of 'canceling each other out' and leaving finite mass bosons only." He goes on: "It is not at all clear that the way for a Sakurai theory [with baryon number as a gauge symmetry] is equally uncluttered. The only mechanism suggested by the present work (of course, we have not discussed non-Abelian gauge groups) for giving the gauge field mass is the degenerate vacuum type of theory, in which the original symmetry is not manifest in the observable domain. Therefore, it needs to be demonstrated that the necessary conservation laws can be maintained." In other words, as one would say today, if baryon number is gauged and spontaneously broken, then baryon number will not be conserved in nature.

And let us look at the end of the paper (fig. 5): "I should like to close with one final remark on the Goldstone theorem. This theorem was initially conjectured, one presumes, because of the solid-state analogs, via the work of Nambu and Anderson" (the reference here is to Nambu's work on spontaneously broken chiral symmetry and to Anderson's paper whose title page appears in fig. 4). He goes on to give various examples, both old (spin waves and phonons) and new (superconductors and superfluids). And then he writes, "It is noteworthy that in most of these cases, upon closer examination, the Goldstone bosons do indeed become tangled up with Yang-Mills gauge bosons and do not in any true sense really have zero mass. Superconductivity is a familiar example, but a similar phenomenon happens with phonons; when the phonon frequency is as low

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I should like to close with one final remark on the Goldstone theorem. This theorem was initially conjectured, one presumes, because of the solid-state analogs, via the work of Nambu[10] and of Anderson.[11] The theorem states, essentially, that if the Lagrangian possesses a continuous symmetry group under which the ground or vacuum state is not invariant, that state is, therefore, degenerate with other ground states. This implies a zero-mass boson. Thus, the solid crystal violates translational and rotational invariance, and possesses phonons; liquid helium violates (in a certain sense only, of course) gauge invariance, and possesses a longitudinal phonon; ferro-magnetism violates spin rotation symmetry, and possesses spin waves; superconductivity violates gauge invariance, and would have a zero-mass collective mode in the absence of long-range Coulomb forces.

It is noteworthy that in most of these cases, upon closer examination, the Goldstone bosons do indeed become tangled up with Yang-Mills gauge bosons and, thus, do not in any true sense really have zero mass. Superconductivity is a familiar example, but a similar phenomenon happens with phonons; when the phonon frequency is as low as the gravitational plasma frequency, $(4\pi G\rho)^{1/2}$ (wavelength$\sim 10^4$ km in normal matter) there is a phonon-graviton interaction: in that case, because of the peculiar sign of the gravitational interaction, leading to instability rather than finite mass.[12] Utiyama[13] and Feynman have pointed out that gravity is also a Yang-Mills field. It is an amusing observation that the three phonons plus two gravitons are just enough components to make up the appropriate tensor particle which would be required for a finite-mass graviton.

Spin waves also are known to interact strongly with magnetostatic forces at very long wavelengths,[14] for rather more obscure and less satisfactory reasons. We conclude, then, that the Goldstone zero-mass difficulty is not a serious one, because we can probably cancel it off against an equal Yang-Mills zero-mass problem. What is not clear yet, on the other hand, is whether it is possible to describe a truly strong conservation law such as that of baryons with a gauge group and a Yang-Mills field having finite mass.

I should like to thank Dr. John R. Klauder for valuable conversations and, particularly, for correcting some serious misapprehensions on my part, and Dr. John G. Taylor for calling my attention to Schwinger's work.

[12] J. H. Jeans, Phil. Trans. Roy. Soc. London **101**, 157 (1903).
[13] R. Utiyama, Phys. Rev. **101**, 1597 (1956); R. P. Feynman (unpublished).
[14] L. R. Walker, Phys. Rev. **105**, 390 (1957).

FIGURE 5. The concluding part of Anderson's 1962 paper on gauge symmetry breaking.

as the gravitational plasma frequency, $(4\pi G)^{1/2}$ (wavelength $\sim 10^4$ km in normal matter) there is a phonon-graviton interaction: in that case, because of the peculiar sign of the gravitational interaction, leading to instability rather than finite mass. Utiyama and Feynman have pointed out that gravity is also a Yang-Mills field. It is an amusing observation that three phonons plus two gravitons are just enough components to make up the appropriate tensor particle which would be required for a finite-mass graviton." So the answer to the question of who first tried to discuss a gravitational analog of gauge symmetry breaking is that Anderson did.

What happened next? In 1964, Peter Higgs wrote two papers on gauge invariance with massive vector particles in relativistic physics. Like Anderson, but unlike Schwinger, his starting point is spontaneous breaking of symmetry. This was a few years after Goldstone's theorem and particle physicists were more familiar with this concept. In the first paper, he explains somewhat abstractly, in a language similar to Schwinger's, why Goldstone's theorem is not valid in the case of a gauge symmetry. Higgs's paper that really had greater impact was the second one in which he described a concrete (and weakly coupled) model that everyone could understand and also introduced the Higgs particle. Let us take a look at this paper (fig. 6).

Higgs explains at the outset that the phenomenon of a gauge boson acquiring a mass via symmetry breaking "is just the relativistic analog of the plasmon phenomenon to which Anderson has

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BROKEN SYMMETRIES AND THE MASSES OF GAUGE BOSONS

Peter W. Higgs
Tait Institute of Mathematical Physics, University of Edinburgh, Edinburgh, Scotland
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In a recent note[1] it was shown that the Goldstone theorem,[2] that Lorentz-covariant field theories in which spontaneous breakdown of symmetry under an internal Lie group occurs contain zero-mass particles, fails if and only if the conserved currents associated with the internal group are coupled to gauge fields. The purpose of the present note is to report that, as a consequence of this coupling, the spin-one quanta of some of the gauge fields acquire mass; the longitudinal degrees of freedom of these particles (which would be absent if their mass were zero) go over into the Goldstone bosons when the coupling tends to zero. This phenomenon is just the relativistic analog of the plasmon phenomenon to which Anderson[3] has drawn attention: that the scalar zero-mass excitations of a superconducting neutral Fermi gas become longitudinal plasmon modes of finite mass when the gas is charged.

The simplest theory which exhibits this behavior is a gauge-invariant version of a model used by Goldstone[2] himself: Two real[4] scalar fields φ_1, φ_2 and a real vector field A_μ interact through the Lagrangian density

$$L = -\tfrac{1}{2}(\nabla\varphi_1)^2 - \tfrac{1}{2}(\nabla\varphi_2)^2 - V(\varphi_1^2+\varphi_2^2) - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}, \qquad (1)$$

about the "vacuum" solution $\varphi_1(x)=0$, $\varphi_2(x)=\varphi_0$:

$$\partial^\mu\{\partial_\mu(\Delta\varphi_1) - e\varphi_0 A_\mu\} = 0, \qquad (2a)$$

$$\{\partial^2 - 4\varphi_0^2 V''(\varphi_0^2)\}(\Delta\varphi_2) = 0, \qquad (2b)$$

$$\partial_\nu F^{\mu\nu} = e\varphi_0\{\partial^\mu(\Delta\varphi_1) - e\varphi_0 A^\mu\}. \qquad (2c)$$

Equation (2b) describes waves whose quanta have (bare) mass $2\varphi_0\{V''(\varphi_0^2)\}^{1/2}$; Eqs. (2a) and (2c) may be transformed, by the introduction of new variables

$$B_\mu = A_\mu - (e\varphi_0)^{-1}\partial_\mu(\Delta\varphi_1),$$
$$G_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu = F_{\mu\nu}, \qquad (3)$$

into the form

$$\partial_\mu B^\mu = 0, \quad \partial_\nu G^{\mu\nu} + e^2\varphi_0^2 B^\mu = 0. \qquad (4)$$

Equation (4) describes vector waves whose quanta have (bare) mass $e\varphi_0$. In the absence of the gauge field coupling ($e=0$) the situation is quite different: Equations (2a) and (2c) describe zero-mass scalar and vector bosons, respectively. In passing, we note that the right-hand side of (2c) is just the linear approximation to the conserved current: It is linear in the vector potential,

FIGURE 6. Peter Higgs's second paper on gauge symmetry breaking, in which he introduced in eqn. (1) what particle physicists know as the abelian Higgs model.

drawn attention: that the scalar zero-mass excitations of a superconducting neutral Fermi gas become longitudinal plasmon modes of finite mass when the gas is charged." He then goes on, in the next paragraph (bottom left in fig. 6), to write down his model.

Higgs's model was simply a relativistic version of the model that Landau and Ginzburg had introduced to describe superconductivity. (Neither Higgs nor any of the authors I have mentioned cited Landau and Ginzburg. Higgs describes his model as a gauge-invariant version of a model of Goldstone.) The Landau-Ginzburg model can be deduced from the action

$$I = \int \mathrm{d}^3x\,\mathrm{d}t\,\left(-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\bar{\Phi}\frac{D}{Dt}\Phi - \frac{1}{2m}\sum_i\left|\frac{D\Phi}{Dx^i}\right|^2 - \lambda(\bar{\Phi}\Phi - a^2)^2\right).$$

Higgs's model, which particle physicists call the abelian Higgs model, is the same thing (apart from minor rescalings) with the kinetic energy of the scalar field made relativistic

$$I = \int \mathrm{d}^3x\,\mathrm{d}t\,\left(-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + D_\mu\bar{\Phi}D^\mu\Phi - \lambda(\bar{\Phi}\Phi - a^2)^2\right).$$

The models are the same except that the abelian Higgs model is quadratic rather than linear in time derivatives.

When Dirac (for spin 1/2) or Klein and Gordon (for spin 0) made the Schrodinger equation relativistic, they introduced an extra degree of freedom (the antiparticle). Something somewhat similar happens in making the Landau-Ginzburg model relativistic. There is an extra degree of freedom because Φ and $\bar{\Phi}$ become independent rather than being canonically conjugate, as they are in Landau-Ginzburg theory. In Landau-Ginzburg theory, if we ignore the gauge fields, there is one spin 0 particle, the Goldstone boson, but if we include the gauge fields, then – as Anderson had explained in the more sophisticated context of BCS theory – it becomes part of a massive spin 1 particle. In the abelian Higgs model, there is a second and massive spin 0 mode – this is the fluctuation in the magnitude of Φ, which is now usually called the Higgs particle.

Actually, although there is not quite a Higgs particle in usual models of superconductivity – or in superconducting phenomenology – there is a close cousin. In a superconductor, there are two characteristic lengths, the penetration depth and the coherence length. They are described in the Landau-Ginzburg and BCS models of superconductivity and are measured experimentally. (The difference between a Type I and Type II superconductor has to do with which is bigger.) These are the analogs of the gauge boson mass and the Higgs boson mass in particle physics. Relativistically, the rate at which the field decays in space is related to a particle mass, but nonrelativistically there is no reason for this to happen, and in the Landau-Ginzburg model it doesn't, in the case of the correlation length. The Landau-Ginzburg model and the abelian Higgs model are completely equivalent for static phenomena since they coincide once one drops the time derivatives.

Similar ideas were developed by others at roughly the same time as Higgs. We will just take a quick look. The paper of Englert and Brout is in fig. 7. This paper is notable for considering symmetry breaking in non-abelian gauge theory, while previous authors had considered the abelian case, sometimes saying that this was for simplicity. "The importance of this problem," they say, "lies in the possibility that strong-interaction physics originates from massive gauge fields coupled to a system of conserved currents," for which they refer to Sakurai. Soon after was the paper of Guralnik, Hagen, and Kibble (fig. 8), followed by Migdal and Polyakov (fig. 9). The title of Migdal and Polyakov, "Spontaneous Breakdown of Strong Interaction Symmetry and Absence of Massless Particles," shows that they, too, were thinking of the strong interactions as the arena in which gauge symmetry breaking might play a role.

From here, let us move forward to Kibble in 1966 (fig. 10). After mentioning the example of superconductivity, Kibble writes "The first indication of a similar effect in relativistic theories was provided by the work of Anderson, who showed that the introduction of a long-range field, like the electromagnetic field, might serve to eliminate massless particles from the theory. More recently, Higgs has exhibited a model which shows explicitly how the massless Goldstone bosons are eliminated by coupling the current associated with the broken symmetry to a gauge field." He then goes on to discuss some important details of symmetry breaking in nonabelian gauge theory. He explains how it it is possible to have partial breaking of nonabelian gauge symmetry, with some gauge mesons remaining massless. Like Higgs and some of the others, he does not really say what the physical application is supposed to be, but he does remark that nature has only one massless vector particle – the photon – but various (in some cases approximate) global symmetries. At least this was on the right track.

BROKEN SYMMETRY AND THE MASS OF GAUGE VECTOR MESONS*

F. Englert and R. Brout
Faculté des Sciences, Université Libre de Bruxelles, Bruxelles, Belgium
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It is of interest to inquire whether gauge vector mesons acquire mass through interaction[1]; by a gauge vector meson we mean a Yang-Mills field[2] associated with the extension of a Lie group from global to local symmetry. The importance of this problem resides in the possibility that strong-interaction physics originates from massive gauge fields related to a system of conserved currents.[3] In this note, we shall show that in certain cases vector mesons do indeed acquire mass when the vacuum is degenerate with respect to a compact Lie group.

those vector mesons which are coupled to currents that "rotate" the original vacuum are the ones which acquire mass [see Eq. (6)].

We shall then examine a particular model based on chirality invariance which may have a more fundamental significance. Here we begin with a chirality-invariant Lagrangian and introduce both vector and pseudovector gauge fields, thereby guaranteeing invariance under both local phase and local γ_5-phase transformations. In this model the gauge fields themselves may break the γ_5 invariance leading to a mass for the original Fermi field. We shall show in this case

FIGURE 7. Brout and Englert on gauge symmetry breaking.

GLOBAL CONSERVATION LAWS AND MASSLESS PARTICLES*

G. S. Guralnik,† C. R. Hagen,‡ and T. W. B. Kibble
Department of Physics, Imperial College, London, England
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In all of the fairly numerous attempts to date to formulate a consistent field theory possessing a broken symmetry, Goldstone's remarkable theorem[1] has played an important role. This theorem, briefly stated, asserts that if there exists a conserved operator Q_i such that

$$[Q_i, A_j(x)] = \sum_k t_{ijk} A_k(x),$$

and if it is possible consistently to take $\sum_k t_{ijk} \times \langle 0|A_k|0\rangle \neq 0$, then $A_j(x)$ has a zero-mass particle in its spectrum. It has more recently been observed that the assumed Lorentz invariance essential to the proof[2] may allow one the hope of avoiding such massless particles through the introduction of vector gauge fields and the consequent breakdown of manifest covariance.[3] This, of course, represents a departure from the assumptions of the theorem, and a limitation on its applicability which in no way reflects on the general validity of the proof.

In this note we shall show, within the framework of a simple soluble field theory, that it is possible consistently to break a symmetry (in the sense that $\sum_k t_{ijk}\langle 0|A_k|0\rangle \neq 0$) without requiring that $A(x)$ excite a zero-mass particle. While this result might suggest a general procedure for the elimination of unwanted massless bosons, it will be seen that this has been accomplished by giving up the global conservation law usually

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FIGURE 8. Guralnik, Hagen, and Kibble on gauge symmetry breaking.

The next milestone, of course, was that in 1967-8, Weinberg and Salam actually found what spontaneous gauge symmetry breaking is good for in particle physics. (Their model was a gauge-invariant refinement of an earlier model by Glashow. That model had W and Z mesons, but lacked the relationship between their masses and couplings that follows from the spontaneous symmetry breaking mechanism introduced by Weinberg and Salam.) However, since we have already looked at quite a few original papers, let us jump ahead to Weinberg's Nobel Prize address in 1979.

In the passage copied in fig. 11, Weinberg explains quite vividly how – like everyone else in the 1960's, it seems – he started by assuming that gauge symmetry breaking was supposed to be applied to the strong interactions. His detailed explanation actually makes interesting reading. To

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SPONTANEOUS BREAKDOWN OF STRONG INTERACTION SYMMETRY AND THE ABSENCE OF MASSLESS PARTICLES

A. A. MIGDAL and A. M. POLYAKOV

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The occurrence of massless particles in the presence of spontaneous symmetry breakdown is discussed. By summing all Feynman diagrams, one obtains for the difference of the mass operators $M_a(p) - M_b(p)$ of particles a and b belonging to a supermultiplet an equation which is identical to the Bethe-Salpeter equation for the wave function of a scalar bound state of vanishing mass (a "zeron") in the annihilation channel $\bar{a}b$ of the corresponding particles. It is shown that if symmetry is spontaneously violated in a Yang–Mills type theory involving vector mesons, the zerons interact only with virtual particles and therefore unobservable. On the other hand, the vector mesons acquire a mass in spite of the generalized gauge invariance. It is shown in Appendices A and B that the asymmetrical solution corresponds to a minimal energy of the vacuum and that C-invariance of the solution implies strangeness conservation for it.

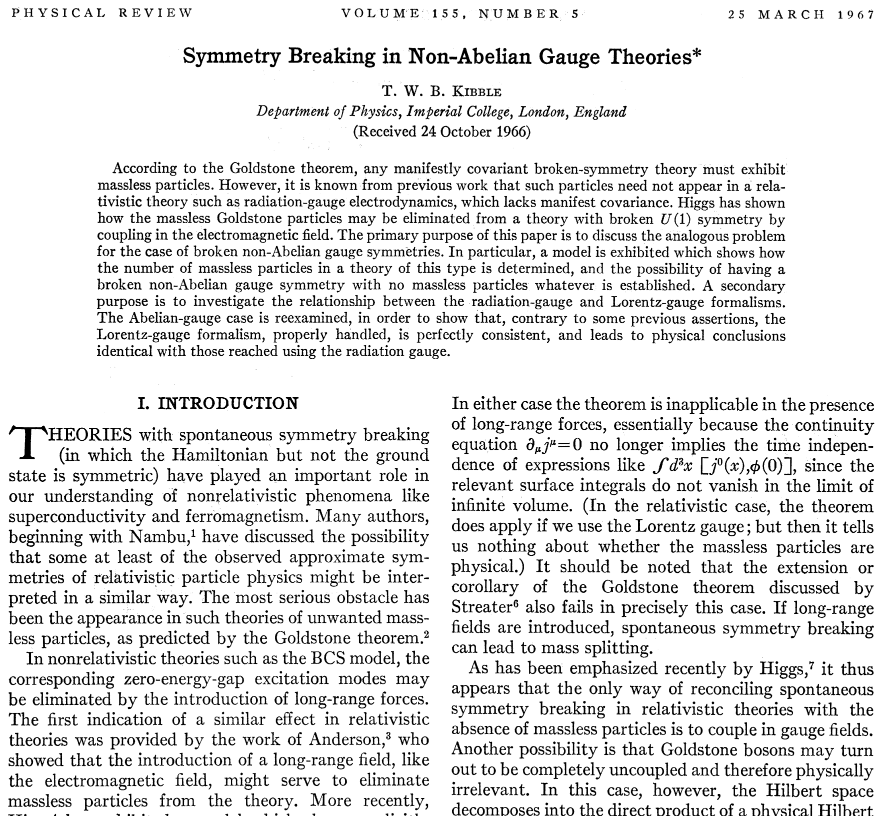
1. INTRODUCTION

SPONTANEOUS symmetry breakdown related to an instability of the system under consideration with respect to an infinitesimally weak asymmetric perturbation is often encountered in quantum statistical mechanics (ferromagnetism, superconductivity, etc.). A large number of such examples has been analyzed in Bogolyubov's review.[1]

FIGURE 9. Migdal and Polyakov on gauge symmetry breaking.

help the reader understand this passage, I will make the following remarks. If baryon number is a gauge symmetry, what is the gauge meson? The lightest hadronic particle of spin 1 with the appropriate quantum numbers is the ω meson, or the ϕ meson if one includes strange particles. So one might think of one of those as a gauge meson. But if baryon number is a gauge symmetry, perhaps isospin symmetry is a gauge symmetry also. In this case, the lightest candidates for the massive gauge particles are the ρ mesons. But bearing in mind that isospin symmetry is part of a spontaneously broken $SU(2) \times SU(2)$ chiral symmetry, perhaps there is also an axial vector triplet of massive gauge mesons; the A_1 is the lightest candidate. All this is quite alien to present-day thinking, and as Weinberg explains, there were a lot of problems: massless ρ mesons, or no pions, or explicit (rather than spontaneous) breaking of gauge invariance and therefore no renormalizability, depending on what assumptions he made.

Then enlightenment dawns. Weinberg explains that "At some point in the fall of 1967, I think while driving to my office at M.I.T., it occurred to me that I had been applying the right ideas to the wrong problem. It is not the ρ mesons that is massless: it is the photon. And its partner is not the A_1, but the massive intermediate boson, which since the time of Yukawa had been suspected to be the mediator of the weak interactions. The weak and electromagnetic interactions could then be described in a unified way in terms of an exact but spontaneously broken gauge symmetry.... And



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Symmetry Breaking in Non-Abelian Gauge Theories*

T. W. B. KIBBLE
Department of Physics, Imperial College, London, England
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According to the Goldstone theorem, any manifestly covariant broken-symmetry theory must exhibit massless particles. However, it is known from previous work that such particles need not appear in a relativistic theory such as radiation-gauge electrodynamics, which lacks manifest covariance. Higgs has shown how the massless Goldstone particles may be eliminated from a theory with broken $U(1)$ symmetry by coupling in the electromagnetic field. The primary purpose of this paper is to discuss the analogous problem for the case of broken non-Abelian gauge symmetries. In particular, a model is exhibited which shows how the number of massless particles in a theory of this type is determined, and the possibility of having a broken non-Abelian gauge symmetry with no massless particles whatever is established. A secondary purpose is to investigate the relationship between the radiation-gauge and Lorentz-gauge formalisms. The Abelian-gauge case is reexamined, in order to show that, contrary to some previous assertions, the Lorentz-gauge formalism, properly handled, is perfectly consistent, and leads to physical conclusions identical with those reached using the radiation gauge.

I. INTRODUCTION

THEORIES with spontaneous symmetry breaking (in which the Hamiltonian but not the ground state is symmetric) have played an important role in our understanding of nonrelativistic phenomena like superconductivity and ferromagnetism. Many authors, beginning with Nambu,[1] have discussed the possibility that some at least of the observed approximate symmetries of relativistic particle physics might be interpreted in a similar way. The most serious obstacle has been the appearance in such theories of unwanted massless particles, as predicted by the Goldstone theorem.[2]

In nonrelativistic theories such as the BCS model, the corresponding zero-energy-gap excitation modes may be eliminated by the introduction of long-range forces. The first indication of a similar effect in relativistic theories was provided by the work of Anderson,[3] who showed that the introduction of a long-range field, like the electromagnetic field, might serve to eliminate massless particles from the theory. More recently,

In either case the theorem is inapplicable in the presence of long-range forces, essentially because the continuity equation $\partial_\mu j^\mu=0$ no longer implies the time independence of expressions like $\int d^3x\, [j^0(x),\phi(0)]$, since the relevant surface integrals do not vanish in the limit of infinite volume. (In the relativistic case, the theorem does apply if we use the Lorentz gauge; but then it tells us nothing about whether the massless particles are physical.) It should be noted that the extension or corollary of the Goldstone theorem discussed by Streater[6] also fails in precisely this case. If long-range fields are introduced, spontaneous symmetry breaking can lead to mass splitting.

As has been emphasized recently by Higgs,[7] it thus appears that the only way of reconciling spontaneous symmetry breaking in relativistic theories with the absence of massless particles is to couple in gauge fields. Another possibility is that Goldstone bosons may turn out to be completely uncoupled and therefore physically irrelevant. In this case, however, the Hilbert space decomposes into the direct product of a physical Hilbert

FIGURE 10. Kibble on symmetry breaking in nonabelian gauge theory.

this theory would be renormalizable like quantum electrodynamics because it is gauge invariant like quantum electrodynamics."

I have been asked whether Weinberg and Salam were the first to use gauge symmetry breaking to give masses to particles other than gauge bosons. They were the first to generate masses for leptons in this way. For strong interactions, matters are more complicated. The modern understanding is that hadron masses come partly from dynamical effects of QCD and partly from the bare masses of quarks and leptons. (The masses of protons, neutrons, and pions come mostly from the QCD effects while heavier hadrons containing charm or bottom quarks get mass mostly from the quark bare masses.) In the modern understanding, it is the quark bare masses, not the part of the hadron masses coming from QCD effects, that result from gauge symmetry breaking and the coupling to the Higgs particle. Such a clear statement was however not possible until QCD was put in its modern form in 1973, enabling the full formulation of the Standard Model. From a modern point of view, earlier attempts to connect hadron masses to gauge symmetry breaking (as opposed to spontaneous breaking of global chiral symmetries) mostly did not focus on the right part of the problem.

The emergence of the Standard Model brings us to the modern era. I will conclude this talk by sketching briefly a few of the subsequent developments. First we will talk about the strong

CONCEPTUAL FOUNDATIONS OF THE UNIFIED THEORY OF WEAK AND ELECTROMAGNETIC INTERACTIONS

Nobel Lecture, December 8, 1979

by STEVEN WEINBERG

Lyman Laboratory of Physics Harvard University and Harvard-Smithsonian Center for Astrophysics, Cambridge, MASS., USA.

Now, back to 1967. I had been considering the implications of the broken SU(2) x SU(2) symmetry of the strong interactions, and I thought of trying out the idea that perhaps the SU(2) x SU(2) symmetry was a "local," not merely a "global," symmetry. That is, the strong interactions might be described by something like a Yang-Mills theory, but in addition to the vector ρ mesons of the Yang-Mills theory, there would also be axial vector Al mesons. To give the ρ meson a mass, it was necessary to insert a common ρ and Al mass term in the Lagrangian, and the spontaneous breakdown of the SU(2) x SU(2) symmetry would then split the ρ and Al by something like the Higgs mechanism, but since the theory would not be gauge invariant the pions would remain as physical Goldstone bosons. This theory gave an intriguing result, that the A1/ρ mass ratio should be $\sqrt{2}$, and in trying to understand this result without relying on perturbation theory, I discovered certain sum rules, the "spectral function sum rules," [23] which turned out to have variety of other uses. But the SU(2) x SU(2) theory was not gauge invariant, and hence it could not be renormalizable, [24] so I was not too enthusiastic about it. [25] Of course, if I did not insert the ρ-A1 mass term in the Lagrangian, then the theory would be gauge invariant and renormalizable, and the Al would be massive. But then there would be no pions and the ρ mesons would be massless, in obvious contradiction (to say the least) with observation.

FIGURE 11. A passage from Steve Weinberg's Nobel Prize Lecture in 1979.

interactions. Since the discovery of asymptotic freedom in 1973, we describe the strong interactions via an *unbroken* non-abelian gauge theory with gauge group $SU(3)$, coupled to quarks. The $SU(3)$ gauge symmetry is definitely unbroken, so at first sight it looks like spontaneous breaking of gauge symmetry turned out to be the wrong idea for the strong interactions.

There is a mystery, however, in QCD: why don't we see the quarks? Experiment and computer simulations both seem to show that the quarks are "confined," that the energy grows indefinitely if one tries to separate a quark from an antiquark. Confinement is quite a surprise and I would say that we still do not fully understand it today. However it was realized in the 1970's that superconductivity comes to the rescue again, giving an understandable explanation of how confinement can happen. An isolated magnetic monopole would have infinite energy in a superconductor because of the Meissner effect. As sketched in fig. 12, a monopole is a source of magnetic flux, but the Meissner effect would cause this flux to be compressed into an Abrikosov-Gorkov flux tube, with an energy proportional to its length.

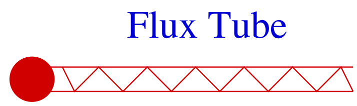

FIGURE 12. In a superconductor, a magnetic monopole (small ball) is the source of a flux tube. As a result, its energy grows linearly with the size of the system.

The best qualitative understanding that we have of confinement today is to say that it involves a "dual" to the Meissner effect, where this "duality" somehow generalizes to nonabelian gauge theory the symmetry of Maxwell's equations that exchanges electric and magnetic fields. Electric charges – quarks – are then confined by a "dual Meissner effect." We do not fully understand this in the case of QCD, but by now we know various situations in four-dimensional gauge theories in which something like this happens.

Now we come to the weak interactions. The original Weinberg-Salam model was based on a weakly-coupled picture with an elementary Higgs field – an elaboration of the Landau-Ginzburg and Higgs models to include nonabelian gauge symmetry and leptons (and later quarks). But many physicists for decades have wondered if the analogy with superconductivity is even stronger – if the breakdown of the electroweak gauge symmetry involves something more like the BCS mechanism of superconductivity.

There have been numerous motivations, and of course different physicists have had different motivations at different times. Some simply suspected that the analogy between the weak interactions and superconductivity would turn out to be even closer. Some considered the model with an elementary scalar field to be arbitrary and inelegant. Another motivation for some was the fact that the Standard Model is not predictive for lepton and quark masses (and "mixing angles"). Each mass is a free parameter, determined by the strength of the coupling of the Higgs field to a given quark or lepton. Maybe a model of "dynamical symmetry breaking," more like the BCS mechanism, would give a more predictive model.

Perhaps the most compelling motivation came from the "hierarchy problem." Although the electroweak gauge theory with a Higgs field is renormalizable, there is a puzzle about it. In the action describing the Higgs field

$$\int \mathrm{d}^4x \left(D_\mu \bar{\Phi} D^\mu \Phi - \lambda(|\Phi|^2 - a^2)^2 \right),$$

the parameter a^2, which determines the mass scale of weak interactions, is a "relevant" parameter in the renormalization group sense. Generic ideas of renormalization theory suggest that a^2 should be in order of magnitude as large as the largest mass scale of the theory – probably the mass scale of gravity or of grand unification of some sort, but anyway much bigger than the mass scale of weak interactions. By analogy, in condensed matter physics, unless one tunes a parameter – such as the temperature – one does not see a correlation length much longer than the lattice spacing. Why the electroweak length scale is so much bigger than the particle physics analog of the lattice spacing is the "hierarchy problem."

There is no problem writing down a model that replaces the Higgs field with a pairing mechanism (involving a new "technicolor" gauge symmetry with "techniquarks") and solves the hierarchy problem. There is even an immediate success: such a model can easily reproduce a relationship between the W and Z masses and the weak mixing angle that was one of the early triumphs of the Standard Model. However, a serious problem was well-recognized in the late 1970's: one can argue that the way the Standard Model gives quark and lepton masses is inelegant and unpredictive, but at least it works. Simple models of "dynamical electroweak gauge symmetry breaking" have serious problems giving realistic quark and lepton masses. Of course, people found clever fixes but it never looked like a match made in heaven.

Experiment began to weigh in seriously in the 1990's. Neither the Higgs particle nor the new particles required by "dynamical" models were discovered. But tests of the Standard Model – especially in e^+e^- annihilation – became precise enough that it was possible to say that the original version of the Standard Model with a simple Higgs field is a better fit than more sophisticated "dynamical" models. There certainly were still fixes, but people had to work harder to find them.

Probably we all know where this story has reached, at least for now. A particle with properties a lot like the Higgs particle of the Standard Model was found a year ago with a mass around 125 GeV. It looks like the electroweak scale is weakly coupled, as is possible in part because of Anderson's insights about gauge symmetry breaking in 1962. But the hierarchy problem is still with us. "Dynamical" models that tried to solve it have not been confirmed, and weakly coupled models – notably based on supersymmetry – that tried to solve it have also not yet been confirmed. I will just end with a question: When the LHC gets to higher energies in 2015, will this situation persist or will it be resolved?