Quantum Mechanics
and
Atomic Physics

Monday & Wednesday 1:40-3:00 SEC 209
http://www.physics.rutgers.edu/ugrad/361

Sean Oh
ohsean@physics.rutgers.edu
Serin W121
Welcome to the World of Quantum Mechanics!

(Sept. 1) Come check this website on a regular basis.
Syllabus

Reading assignments are from Reed sections. You should complete the reading assignment before the lecture takes place. Lecture notes will be posted on this webpage after each class.

<table>
<thead>
<tr>
<th>WEEK</th>
<th>DATES</th>
<th>LECTURE TOPICS</th>
<th>READING</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Wed 9/7</td>
<td>Introduction/Foundation I</td>
<td>1.1-1.2</td>
</tr>
<tr>
<td></td>
<td>Thur 9/8</td>
<td>Foundation II</td>
<td>1.2-1.4</td>
</tr>
<tr>
<td>2</td>
<td>Mon 9/12</td>
<td>Schrödinger Equation I</td>
<td>2.1-2.2</td>
</tr>
<tr>
<td></td>
<td>Wed 9/14</td>
<td>Schrödinger Equation II</td>
<td>2.3-2.4</td>
</tr>
<tr>
<td>3</td>
<td>Mon 9/19</td>
<td>Potential Wells I</td>
<td>3.1-3.2</td>
</tr>
<tr>
<td></td>
<td>Wed 9/21</td>
<td>Potential Wells II</td>
<td>3.3-3.6</td>
</tr>
<tr>
<td>4</td>
<td>Mon 9/26</td>
<td>Potential Barriers and Scattering</td>
<td>5.7-5.10</td>
</tr>
<tr>
<td></td>
<td>Wed 9/28</td>
<td>Operators and Expectation Values</td>
<td>4.1-4.2</td>
</tr>
<tr>
<td>5</td>
<td>Mon 10/3</td>
<td>Uncertainty Principle and Commutators</td>
<td>4.3-4.5</td>
</tr>
<tr>
<td></td>
<td>Wed 10/5</td>
<td>Orthogonality and Superposition</td>
<td>4.6-4.9</td>
</tr>
<tr>
<td>6</td>
<td>Mon 10/10</td>
<td>Harmonic Oscillator I</td>
<td>5.1-5.3</td>
</tr>
<tr>
<td></td>
<td>Wed 10/12</td>
<td>Harmonic Oscillator II</td>
<td>5.4-5.6</td>
</tr>
<tr>
<td>7</td>
<td>Mon 10/17</td>
<td>Review for Midterm Exam</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Wed 10/19</td>
<td>Midterm Exam</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>Mon 10/24</td>
<td>Schrödinger’s Equation in 3D</td>
<td>6.1-6.2</td>
</tr>
<tr>
<td></td>
<td>Wed 10/26</td>
<td>Angular Momentum I</td>
<td>6.3-6.4</td>
</tr>
<tr>
<td>9</td>
<td>Mon 10/31</td>
<td>Angular Momentum II</td>
<td>6.5</td>
</tr>
<tr>
<td></td>
<td>Wed 11/2</td>
<td>Central Potentials</td>
<td>7.1-7.3</td>
</tr>
<tr>
<td>10</td>
<td>Mon 11/7</td>
<td>Coulomb Potential</td>
<td>7.4</td>
</tr>
<tr>
<td></td>
<td>Wed 11/9</td>
<td>Hydrogen Atom</td>
<td>7.5-7.7</td>
</tr>
<tr>
<td>11</td>
<td>Mon 11/14</td>
<td>Raising and Lowering Operators</td>
<td>8.1</td>
</tr>
<tr>
<td></td>
<td>Wed 11/16</td>
<td>Angular Momentum Quantization and Spin</td>
<td>8.2-8.3</td>
</tr>
<tr>
<td>12</td>
<td>Mon 11/21</td>
<td>Identical Particles and Pauli Exclusion Principle</td>
<td>8.4</td>
</tr>
<tr>
<td></td>
<td>Wed 11/23</td>
<td>Happy Thanksgiving!</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>Mon 11/28</td>
<td>WKB method</td>
<td>9.1-9.2</td>
</tr>
<tr>
<td>14</td>
<td>Mon 12/5</td>
<td>Variational Method</td>
<td>9.4</td>
</tr>
<tr>
<td></td>
<td>Wed 12/7</td>
<td>Time-Dependent Perturbation theory</td>
<td>11.1-11.3</td>
</tr>
<tr>
<td>15</td>
<td>Mon 12/12</td>
<td>GRE problems/Review for Final Exam</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Final Exam</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Wed 12/21</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Textbook(s)

- "Quantum Mechanics" by Bruce Cameron Reed (Jones and Bartlett Publishers, Copyright 2008)
  - I will primarily follow the material in this book
  - I encourage you to read the material prior to attending class
Homework

- The homework will come primarily from the book
  - You will have weekly homework assignments due at the beginning of class on Mondays
  - First homework is due on September 19
Grades

- Your grade will be based on four components:
  - weekly homework (20%)  
  - frequent in-class quizzes (20%) 
  - a midterm exam (20%) 
  - a final exam (40%) 

- Attendance is strongly advised.
  
  *There are no make-ups for missed quizzes.*
Introduction

- Qualitatively, QM can be viewed as a theory that correctly describes the behavior of microscopic material particles but incorporates the predictions of Newtonian mechanics on macroscopic scales.

- We will first review some of the facts that led physicists in the early 20th century to the realization that Newtonian mechanics is invalid in the realm of atomic scale phenomena.
Kirchhoff’s Laws of Spectroscopy

- In the 1850’s, Gustav Kirchhoff observed:
  - Light from hot object through prism $\Rightarrow$ continuous spectrum
  - Light through cool gas $\Rightarrow$ same spectrum but with certain wavelengths of light removed
  - Light emitted by hot gas alone $\Rightarrow$ series of bright lines at certain wavelengths
Thermal Radiation

1. Objects can reflect light
2. Objects can emit light
   a) Thermal (continuous) spectrum: all wavelengths
   b) Line spectrum: discrete wavelengths

- Explaining discrete line spectra posed a challenge to builders of atomic models.
- Continuous spectra also posed challenges to classical physics.
  - All materials emit thermal radiation or “blackbody” radiation
  - Hotter temperatures yield shorter wavelengths
  - Attempts to find a mathematical description of thermal radiation based on classical models failed.
Thermal Radiation, con’t

- There are three themes in thermal radiation that led to quantum mechanics:
  - Wien's displacement law
  - Rayleigh-Jeans’ failed attempt using classical laws
  - Planck’s law of radiation
Wien’s displacement law

- A “blackbody” absorbs all light shining on it
  - No reflection
  - But it emits thermal radiation
- Wien’s Law (In 1893, a general form for blackbody radiation):
  - The main frequency (or color) of the emitted radiation increases as the temperature increases.
    \[ \lambda_{\text{max}} T = 2.8979 \times 10^{-3} \text{ m} \cdot \text{K} \]
    \[ \nu_{\text{max}} / T = 5.88 \times 10^{10} \text{ Hz/K} \]
  - As T increases:
    \( \lambda_{\text{max}} \) decreases / \( \nu_{\text{max}} \) increases
    and the peaks are displaced.
Rayleigh-Jeans Theory and the Ultraviolet Catastrophe

- Rayleigh-Jeans used **classical physics** to derive the energy density $\rho(\nu)$ and $\rho(\lambda)$.

Cavity radiation $\approx$ Blackbody radiation

\[
R(\nu) = \frac{c}{4} \rho(\nu), \quad R(\lambda) = \frac{c}{4} \rho(\lambda)
\]

\[
\rho(\nu) = \frac{8\pi\nu^2kT}{c^3}, \quad \rho(\lambda) = \frac{8\pi kT}{\lambda^4}
\]

- “Ultraviolet Catastrophe”

\[
\lim_{\nu \to \infty} \rho(\nu) = \infty
\]

Classical theory prediction of blackbody radiation seriously discrepant with experimental data!
Max Planck’s hypothesis (1900):

- Energy of light at a given $\nu$ is quantized:
  \[ E = n\hbar \nu, \quad n = 1, 2, 3, \ldots \]
  \[ h = \text{Planck's constant} = 6.63 \times 10^{-34} \text{ J} \cdot \text{s} \]

- Thermal radiation, even at a single temperature, occurs at a wide range of frequencies. How much of each frequency is given by Planck’s law of radiation.

- Energy density (energy/unit frequency or energy/unit wavelength):
  \[ \rho(\nu) = \frac{8\pi\nu^2}{c^3} \frac{h \nu}{e^{h \nu / kT} - 1} \]
  \[ \rho(\lambda) = \frac{8\pi\hbar c}{\lambda^5} \frac{1}{e^{hc / \lambda kT} - 1} \]

- This ushered in QM!
Example

Show that the spectral energy density is proportional to $T$ in the low-frequency or so-called classical region.

\[
\rho(v) = \frac{8\pi \gamma^2}{c^3} \frac{hv}{e^{hv/kT} - 1}
\]

\[
\frac{1}{e^{hv/kT} - 1} \approx \frac{kT}{hv} \quad \text{for} \quad \frac{hv}{kT} \ll 1
\]

\[
\rho(v) \approx \frac{8\pi hv^3}{c^3} \cdot \frac{kT}{hv} = \frac{8\pi \gamma^2}{c^3} kT
\]

\[
\rho(v) \propto T
\]

Rayleigh-Jeans result!
The Photoelectric Effect

- Planck’s hypothesis remained dormant until 1905 when Einstein adapted it to explain the photoelectric effect.

- Light shines on metal plate and dislodges electrons which travel to cathode plate.

- Circuit has photocurrent \( i \). Apply retarding potential. Stopping potential \( V_0 \) is needed to kill photocurrent.
Photoelectric Effect

\[ eV_0 = \frac{1}{2} m v_{\text{max}}^2 \]

- But other features of the photoelectric effect made no sense ....
Photoelectric Effect

- $V_0$ vs. $\nu$:
  - Why are the lines parallel?
  - For each substance (Cu, K, Cs) why is there a threshold frequency below which no photoemission occurs?
Einstein’s Explanation (1905)

- Light consists of individual particles ("photons") each of energy \( E = h \nu = \frac{hc}{\lambda} \)
- Photoelectric effect: each photon liberates one electron: one-on-one process.
- Intensity of light \( I = Nh\nu \) (in \( W/m^2 \)) and \( N = \# \) photons/area/second
- Energy conservation:
  \[ h \nu = \omega_0 + \frac{1}{2} m v_{\text{max}}^2 = \omega_0 + eV_0 \]
- "work function" \( \omega_0 = \) binding energy of the least tightly bound electrons
Einstein’s explanation

- These plots are of $eV_0 = h\nu - \omega_0$

- All lines have same slope $h$! But $\omega_0$ depends on the specific metal and $\omega_0 = h\nu_{\text{Threshold}}$
Light-Quanta or Photons

- Wave model is necessary to describe the propagation of light, especially interference.
- Particle model is necessary to describe the interactions of light and matter.
  - The two are contradictory!
  - Wave-particle duality!
- Photon energy: \( E = h \nu = \frac{hc}{\lambda} \)
  - \( m_\gamma = 0 \) and \( v = c \)
  \[
  E = \gamma mc^2 = \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{0}{0} = \text{finite!}
  \]
  - But we can use \( E^2 = (pc)^2 + (mc^2)^2 \) since \( m_\gamma = 0 \)
  \[
  E = pc \text{ for photons}
  \]
- Photon momentum: \( p = \frac{E}{c} = \frac{h}{\lambda} \)
Units

- 1 Angstrom \( \text{Å} \) = \( 10^{-10} \) m

\[
h = (6.63 \times 10^{-34} \text{ J} \cdot \text{s})(\frac{1\text{eV}}{1.6 \times 10^{-19} \text{ J}})(\frac{3 \times 10^8 \text{ m/s}}{c})(\frac{1\text{ Å}}{10^{-10} \text{ m}}) = 12400 \text{Å} \frac{\text{eV}}{c}
\]
Example

Light of $\lambda = 2500$ Angstroms shines on Potassium ($\omega_0=2.20\,\text{eV}$).

a) Find the maximum kinetic energy of the photoelectrons.

b) Find the stopping potential.
a) Max kinetic energy

\[
E = h \nu = \frac{hc}{\lambda} = \frac{(12400 \text{ Å} \cdot \frac{eV}{\text{Å}})}{2500 \text{ Å}} = 4.96eV
\]

Photoelectric equation: \( E_\gamma = \omega_0 + K_{\text{max}} \)

\[ \Rightarrow K_{\text{max}} = 4.96 - 2.20 = 2.76eV \]

b) \( K_{\text{max}} = eV_0 \Rightarrow V_0 = 2.76 \text{ Volts} \)
Summary

- The need to introduce the hypothesis that some physical quantities are quantized grew out of the inability of classical physics to provide adequate understanding of phenomena such as thermal radiation.
  - Planck’s blackbody radiation formula
  - Explained phenomena such as blackbody radiation and the photoelectric effect.
  - Light regarded as stream of particles, photons.

- Next time
  - The Rutherford-Bohr atom
  - de Broglie Matter-Waves

- First homework due on Monday Sept 19! Assignments will show up on the course webpage.