Final Exam Problems

[10 points]

1. Consider electrons arriving at an anode with the probability per unit time $d$. Write down the master equation that governs this stochastic process (Hint: use continuous time formulation). Use the characteristic function method to find $P(n, t|10, 0)$ — the probability that $n$ $\bar{e}$'s arrived at time $t$, given that no $\bar{e}$'s were observed at time $0$. What is the name of this distribution?

[15 points]


   a) Consider a population of $n$ identical individuals, each of which can give birth to an identical offspring at a rate 1 (for simplicity). Write down and solve the mean-field equation for the average population size $\langle n \rangle$ (assume that the process begins with a single individual at $t=0$). Now, write down and solve the stochastic
master equation for \( p_n(t) \) - the prob.
that the population has \( n \) individuals
at time \( t \) \( (p_n(0) = \delta_{n,1}) \).

**Hint:** use the exponential ansatz
\( \sum_{n=1}^{\infty} p_n(t) = A(t) d(t) \) & recall that
\( \sum_{n=1}^{\infty} p_n(t) = 1 \)

Find \( <n> \) & \( \sigma^2 = <n^2> - <n>^2 \).
Is \( <n> \) the same as that obtained
from the mean-field equation?
Compare \( <n> \) & \( \sigma \). Can we neglect
the fluctuations, and if so, when?

b) **Birth and death**
Now consider a population with birth
rate \( \lambda \) and death rate \( \mu \). Carry
out the same calculations as in part a):
find \( <n> \), \( p_n(t) \) [given \( p_n(0) = \delta_{n,1} \)],
\( <n> \) & \( \sigma^2 \) from \( p_n(t) \). Comment
again on the relative size of
fluctuations compared to \( <n> \). Finally,
find the survival prob. \( S(t) = 1 - \rho_0(t) \)
and determine its \( t \to \infty \) limit.

**Hint:** use the exponential ansatz again
but note that \( \sum_{n=0}^{\infty} p_n(t) = 1 \) since
extinction is possible.
\( \sum_{n=1}^{\infty} \Rightarrow p_n(t) = A(t) d^{n-1}(t) \)
[5 points] Using the Max Ent procedure, derive the microcanonical distribution of statistical mechanics.