Final Exam Problems

[10 points]

1. Consider electrons arriving at an anode with the probability per unit time α. Write down the master equation that governs this stochastic process (Hint: use continuous time formulation). Use the characteristic-function method to find $P(n, t \mid 0, 0)$ – the probability that n $\bar{e}$'s arrived at time t, given that no $\bar{e}$'s were observed at time 0. What is the name of this distribution?

[15 points]

2. Consider stochastic birth-death processes. Use continuous time formulation throughout.

a) Birth only. Consider a population of n identical individuals, each of which can give birth to an identical offspring at a rate 1 (for simplicity). Write down and solve a mean-field equation for the average population size $\langle n \rangle$ (assume that the process begins with a single individual at $t=0$). Now, write down and solve the stochastic master equation for $P_n(t)$ - the prob. that the population has n individuals at time t ($P_n(0) = \delta_{n,1}$).

<u>Hint</u>: use the exponentiale ansatz

$\underline{n \geq 1} \Rightarrow P_n(t) = A(t)\, a^{n-1}(t)$ & recall that

$$\sum_{n=1}^{\infty} P_n(t) = 1$$

Find $\langle n \rangle$ and $\sigma^2 = \langle n^2 \rangle - \langle n \rangle^2$.

Is $\langle n \rangle$ the same as that obtained from the mean-field equation?

Compare $\langle n \rangle$ & σ. Can we neglect the fluctuations, and if so, when?

b) <u>Birth and death</u>

Now consider a population with both birth rate λ and death rate μ. Carry out the same calculations as in part a): find $\langle n \rangle$, $P_n(t)$ [given $P_n(0) = \delta_{n,1}$], $\langle n \rangle$ & σ^2 from $P_n(t)$. Comment again on the relative size of fluctuations compared to $\langle n \rangle$. Finally, find the survival prob. $S(t) = 1 - P_0(t)$ and determine its $t \to \infty$ limit.

<u>Hint</u>: use the exponential ansatz again but note that $\sum_{n=0}^{\infty} P_n(t) = 1$ since extinction is possible.

$\underline{n \geq 1} \Rightarrow P_n(t) = A(t)\, a^{n-1}(t)$

[5 points]

3. Using the MaxEnt procedure, derive the microcanonical distribution of statistical mechanics.