1. [Probabilistic generative models]

Generate \( N_1 = 200 \) datapoints from \( N(\mathbf{x} | \tilde{\mu}_1, \Sigma) \) and \( N_2 = 400 \) datapoints from \( N(\mathbf{x} | \tilde{\mu}_2, \Sigma) \), where 
\[ \tilde{\mu}_1 = (2, 2) \quad \text{and} \quad \tilde{\mu}_2 = (-2, -2), \]
\[ \Sigma = \begin{pmatrix} 0.25 & 0 \\ 0 & 0.25 \end{pmatrix} \Rightarrow \delta = 2 \]

Assume that the class-conditional densities are given by
\[
\begin{align*}
    p(\mathbf{x} | \mathcal{C}_1) &= N(\mathbf{x} | \tilde{\mu}_1, \Sigma), \\
    p(\mathbf{x} | \mathcal{C}_2) &= N(\mathbf{x} | \tilde{\mu}_2, \Sigma).
\end{align*}
\]

(a) Using exact values of \( \tilde{\mu}_1, \tilde{\mu}_2, \Sigma, N_1, N_2 \), find the exact decision boundary at which \( p(\mathcal{C}_1 \mid \mathbf{x}) = p(\mathcal{C}_2 \mid \mathbf{x}) = 0.5 \).

Plot the class-conditional densities and the exact DB as a heatmap or a contour map.

(b) Estimate \( p(\mathcal{C}_1 \mid D) \), \( p(\mathcal{C}_2 \mid D) \), \( \tilde{\mu}_1, \tilde{\mu}_2, \Sigma \) by ML and find the DB using these estimated values. Add the DB to the plot in (a).
2. [Logistic regression]

Use the dataset from problem 1 and find the DB by logistic regression. Add it to the plot created for problem 1 and comment on whether the DB found by logistic regression corresponds to the other two (exact DB and the ML DB).

3. Bishop 4.19