1. Consider general 3D motion of a mass \( m_1 \) moving without friction on the inside of the cone surface (see Fig.). Another mass, \( m_2 \), is attached by a string of constant length \( L \) to \( m_1 \); \( m_2 \) can only move along the \( z \) axis. Write down the Lagrangian of the system, implementing all the constraints and carefully justifying the choice of generalized coordinates.

Hint: use cylindrical coordinates \((r, \theta, z)\) for \( m_1 \).
2. Consider a particle of mass $m$ moving in a central force field defined by a potential $U(r)$. Assume that $U(r) \leq 0$ and the central force is pointing towards the origin.

(a) Show that the angular momentum of the particle $\mathbf{l}$ (defined with the origin of the force) is conserved.

(b) Assuming that $\mathbf{l} \neq 0$, argue that the particle's motion occurs in the plane which is $\mathbf{l}$. 

(c) Show that the total energy of the particle is given by

$$E = \frac{1}{2} m r^2 + U_{\text{eff}}(r),$$

and find $U_{\text{eff}}(r)$.

(d) Find the condition for the circular orbit (if it exists), and write down an implicit equation for its radius $r_0$.

(e) Find the condition for the circular orbit to be stable.
(f) Consider \( \vec{F} = -\left( \frac{\vec{b}}{r^2} - \frac{\vec{c}}{r^4} \right) \), where \( b > 0, \ c > 0 \) and \( \vec{r} \) is the radial unit vector. Find the range \( 0 \leq r_0 \) for which stable circular orbits are possible. What happens to this range as \( c \to 0 \)? Why? (i.e., explain the difference between \( \vec{F} \) above and \( \vec{F}' = -\frac{\vec{b}}{r^2} \)).