Physics 406
2012

HW#8 solutions

1. Omar Q. 2

Eq. (10.12):

$$H_c(0) \simeq \left(\frac{2 n k_B^2}{\mu_0 E_F} \right)^{1/2} T_c$$

Recall that in 3D,

$$E_F \sim n^{2/3}$$

Thus $H_c(0) \sim n^{1/6} T_c$, the dependence on n is very weak

$H_c(0) \sim T_c$ for any superconductor

3. Omar Q. 6

The binding energy of a Cooper pair (i.e. the energy required to break it up) is roughly $\Delta(T)$, the gap energy. It is easier to break up the Cooper pair as $T \uparrow$ due to thermal fluctuations. Thus we expect $\Delta(T)$ to decrease with $T \uparrow$ until $T = T_c$. At $T \geq T_c$ Cooper pair formation is no longer energetically favorable.

(2.) <u>Omar Q.5</u>

London equation:

$$\vec{B} = -\frac{m}{n_s e^2} \vec{\nabla} \times \vec{J}_s \quad , \text{ where}$$

$$\vec{J}_s = -n_s e \vec{V}_s .$$

$$B_y = -\frac{m}{n_s e^2}\left(\overbrace{\frac{\partial J_{s,x}}{\partial z}}^{=0} - \frac{\partial J_{s,z}}{\partial x}\right) =$$

$$= \frac{m}{n_s e^2} \frac{\partial J_{s,z}}{\partial x} .$$

But $B_y(x) = B_y(0)\, e^{-x/\lambda}$, so that

$$J_{s,z}(x) = \left(\frac{n_s e^2}{m}\right)(-\lambda)\, B_y(x) =$$

$$= -\left(\frac{n_s^2 e^4}{m^2} \frac{m}{\mu_0 n_s e^2}\right)^{1/2} B_y(x) =$$

$$= -\left(\frac{n_s e^2}{\mu_0 m}\right)^{1/2} B_y(x) \equiv -J_{s,z}(0)\, e^{-x/\lambda} .$$

(4) θ Ch. 10, Pr. 1

Solenoid:

$$L \frac{dI}{dt} + IR = 0,$$

$$\frac{dI}{I} = -\underbrace{\frac{R}{L}}_{\Gamma} dt \Rightarrow I(t) = I(0)\, e^{-\Gamma t},$$

where Γ is the damping timescale

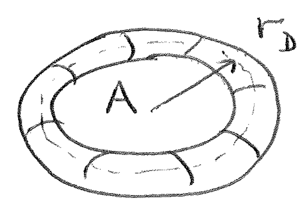
Current damped by 0.01%:

$$\frac{I}{I(0)} = 0.9999 = e^{-\Gamma \tau}, \text{ or}$$

$$\tau = -\frac{\log(0.9999)}{\Gamma}$$

Now, $R = \rho \dfrac{l_w}{A} = \rho \dfrac{l_w}{\pi \left(\frac{d_w}{2}\right)^2}$

$$L = \frac{\Phi}{I} = \frac{\mu_0 N^2 A}{2\pi r_D}$$



$\mu_0 N I = B(2\pi r_D)$, or

N: # turns

$$B = \frac{\mu_0 N I}{2\pi r_D}$$

Then $\Phi = NBA = \dfrac{\mu_0 N^2 A}{2\pi r_D} I$

So, $\underbrace{\dfrac{R}{L}}_{\Gamma} = \dfrac{\rho\, l_w}{\pi \left(\frac{d_w}{2}\right)^2} \times \dfrac{2\pi r_D}{\mu_0 N^2 A}$

Not all #'s are given ⇒ leave in symbolic form

5) θ Ch. 10, Pr. 6

Perfect diamagnetism is considered the more fundamental property of a superconductor.

Indeed, $B=0$ inside a sc ($T<T_c$) does not follow from perfect conductivity:

Ohm's law $\vec{E} = \rho \vec{j}$

$\rho \to 0$ but j finite: $\vec{E} = 0$

Maxwell's equation

$$\vec{\nabla} \times \vec{E} = \frac{\partial \vec{B}}{\partial t} \Rightarrow \frac{\partial \vec{B}}{\partial t} = 0 \text{ since } \vec{E} = 0.$$

Thus a perfect conductor implies that $\vec{B} = \text{const}$, <u>not</u> $\vec{B} = 0$ as observed below T_c (Meissner effect). In other words, just a perfect conductor cannot have a change in magnetic flux after cooling through T_c.

6. Key points on the intermetallic superconductor MgB_2.

a) Original idea: investigation of ternary compounds with 2 light atoms (Mg & B) and one intermediate one (Ti). Expected large DoS $\Rightarrow$ large T_c.

b) Discovered that $T_c \sim 40K$ is actually due to binary compound MgB_2.

c) MgB_2 wires can be made relatively easily $\Rightarrow$ practical applications.

d) Observation of the isotope effect in MgB_2 $\Rightarrow$ role of $\bar{e}$-phonon interactions.

e) Combination of high critical field & low normal-state resistivity make MgB_2 attractive for making sc magnets.

f) Mechanism of sc unknown $\Rightarrow$ two superconducting gaps?