MW #5 solutions

Physics 406

(1.) O Ch. 4, Q. 5

With thermal conduction, $\bar{e}$'s at one end of the sample have more energy than $\bar{e}$'s at the other end. More energetic $\bar{e}$'s diffuse down the T gradient, carrying a net energy flux. On average, there is no particle or charge buildup which would be quite unfavorable energetically.

(2.) O Ch. 4, Q. 8

The Hall constant is defined as

$$R_H \equiv \frac{E_H}{J_x B},$$

where E_H is the Hall field, B is the external magnetic field, and J_x is the current density. Since $J_x \sim N$, ($\bar{e}$ conc'n)

$$R_H \sim \frac{1}{N}.$$

(3.) O Ch. 4, Pr. 10

$$\nu_c = \frac{\omega_c}{2\pi} = 2.8\, B \text{ GHz} \quad \text{for} \quad m^* = m_0,$$

B in kG; the free $\bar{e}$ mass

Then $\nu_c = 24$ GHz gives

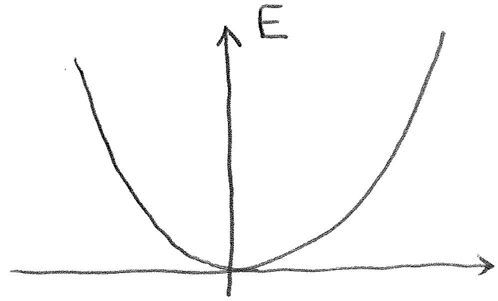
$$B = \frac{24}{2.8} \approx 8.6 \text{ kG}.$$

(4.) O Ch. 5, Q. 2

For a truly free $\bar{e}$,

$$\begin{cases} \psi_k^{(0)} = \frac{1}{\sqrt{L}} e^{ikx}, & \leftarrow \text{1d} \\ E_k^{(0)} = \frac{\hbar^2 k^2}{2m_0} \end{cases}$$

This leads to a single dispersion curve:



"Cutting & pasting" is not justifiable here because there is no periodicity of the lattice:

$$E_k = E_{k+G}, \quad G = \frac{2\pi n}{a},$$

$n = 0, \pm 1, \pm 2, \ldots$

Thus the difference between empty lattice & free space is that we impose symmetry properties in k-space (even though there is no potential) in the former case. These symmetries follow from the translational symmetry of the real lattice.

(5.) θ. Ch. 5, Pr. 12 (1D)

a) $k = \frac{2\pi}{L} n \Rightarrow \underset{\uparrow}{N} = \frac{k}{(2\pi/L)} = \frac{Lk}{2\pi}$

\# states with $\leq k$

$$\frac{dN}{dk} = \frac{L}{2\pi} = \frac{1}{2\pi} \quad \text{ie } L = 1$$

$$g(E) = \frac{dN}{dE} = \frac{dN}{dk}\frac{dk}{dE} = \frac{1/2\pi}{(dE/dk)}$$

b) TB model:

(5.43) in θ

$$E(k) = E_0 + 4\gamma \sin^2\left(\frac{ka}{2}\right)$$

Then

$$\frac{dE}{dk} = 8\gamma \sin\left(\frac{ka}{2}\right)\cos\left(\frac{ka}{2}\right) \times \frac{a}{2} =$$

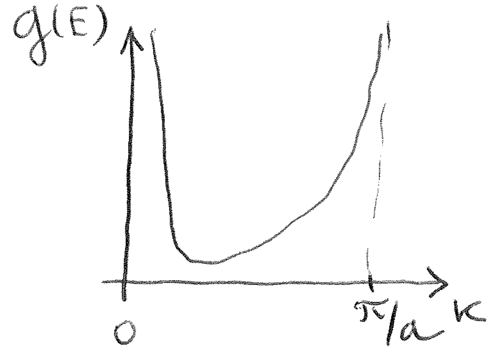
$$= \underline{\underline{2\gamma a \sin(ka)}}.$$

So, $g(E) = \frac{1/2\pi}{2\gamma a \sin(ka)}$

Limits:

$$k \to 0: \quad g(E) \sim \frac{1}{k} \sim \frac{1}{\sqrt{E(k)-E_0}}$$

$$k \to \frac{\pi}{a}: \quad g(E) \to \infty$$



(6.) θ Ch. 5, Pr. 14

a) Free $\bar{e}$ model:

$$n = \underset{\text{spin}}{2} \frac{1}{(2\pi)^3} \frac{4}{3}\pi k_F^3 = \frac{1}{3\pi^2} k_F^3, \text{ or}$$

$$\underline{\underline{k_F = (3\pi^2 n)^{1/3}}}.$$

b) Fermi sphere will touch the face of the 1st BZ when

$k_F = k_i$, where

$\vec{k}_i = \frac{1}{2}\vec{a}$ for fcc, and

$(\text{or } \frac{1}{2}\vec{b}, \frac{1}{2}\vec{c})$

$$\begin{cases} \vec{a} = \frac{2\pi}{a}(1,-1,1) \\ \vec{b} = \frac{2\pi}{a}(1,1,-1) \\ \vec{c} = \frac{2\pi}{a}(-1,1,1) \end{cases}$$

Then $k_i = \frac{1}{2}\frac{2\pi}{a}\sqrt{1^2+1^2+1^2} = \frac{\sqrt{3}\,\pi}{a}$.

For fcc, the # of atoms is

$\frac{4}{a^3}$ & the # of $\bar{e}$'s is (per unit volume)

$$\frac{4}{a^3}\underbrace{\frac{n}{n_a}}_{\bar{e}\text{-to-atom ratio}}$$

Then

$$\left(3\pi^2 \frac{4}{a^3}\frac{n}{n_a}\right)^{1/3} = \frac{\sqrt{3}\,\pi}{a}, \text{ or}$$

$$\frac{n}{n_a} = \frac{1}{12\cancel{\pi^2}} 3^{3/2}\pi^{\cancel{3}} = \frac{\sqrt{3}\,\pi}{4} = 1.36.$$

c) Zn divalent

Cu monovalent

$$\underset{\uparrow \atop \bar{e}\text{ concentr'n}}{n} = \frac{4}{a^3}\left\{(1-x)\times 1 + \underset{\uparrow \atop \text{Zn fraction}}{x}\times 2\right\} = \frac{4}{a^3}(1+x).$$

$k_F = k_i$ as in (b):

$$(3\pi^2 n)^{1/3} = \frac{\sqrt{3}\,\pi}{a}, \text{ or}$$

$$(12\pi^2)^{1/3}(1+x)^{1/3} = \sqrt{3}\,\pi,$$

$$\left(\frac{12}{\pi}\right)^{1/3}(1+x)^{1/3} = \sqrt{3} \Rightarrow 1+x = \frac{\pi}{12}3\sqrt{3} =$$

$$= \frac{\pi\sqrt{3}}{4} \Rightarrow x = \frac{\sqrt{3}\,\pi}{4} - 1 = 0.36,$$

consistent with (b).

7. $m\left(\frac{dv}{dt}+\frac{v}{\tau}\right)=-eE,$

$$\begin{cases} v=v_0 e^{-i\omega t}, \\ E=E_0 e^{-i\omega t} \end{cases} \text{give}$$

$$m\left(-i\omega v_0+\frac{v_0}{\tau}\right)=-eE_0, \text{ or}$$

$$v_0=\frac{-eE_0/m}{-i\omega+1/\tau}=-\frac{eE_0\tau}{m}\frac{1}{-i\omega\tau+1}=$$

$$=-\frac{eE_0\tau}{m}\frac{1+i\omega\tau}{1+\omega^2\tau^2}.$$

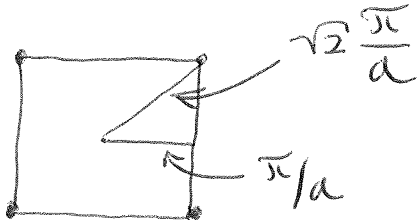
Then

$$j=-nev=\frac{ne^2\tau}{m}E\frac{1+i\omega\tau}{1+\omega^2\tau^2},$$

so that

$$\sigma(\omega)=\underbrace{\sigma_0}_{\frac{ne^2\tau}{m}}\frac{1+i\omega\tau}{1+\omega^2\tau^2}.$$

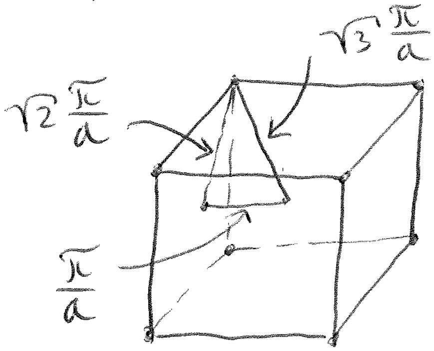
8. a) 2D square lattice



$$k_{corner} = \sqrt{2}\, k_{center}$$

$$E \sim k^2 \Rightarrow E_{corner} = 2\, E_{center}$$

b) 3D sc lattice



$$k_{corner} = \sqrt{3}\, k_{center},$$

$$E_{corner} = 3\, E_{center}$$

c) If $E_{gap} < E_{corner} - E_{center}$,
$\bar{e}$'s will start occupying center states in 2nd BZ rather than corner states in 1st BZ.
If this is the case, divalent elements can become metals rather than insulators.

9. Photonic crystals are devices in which a distribution of refractive indices is chosen such that incoming light waves become standing waves due to refraction & reflection. This makes them analogous to semiconductor devices, except light waves are used instead of $\bar{e}$ waves. Some of the challenges are:

a) identifying suitable materials (with necessary optical properties) & building structures out of them

b) the devices must be able to handle incoming waves coming in all directions

Potential uses: lasers, fiber optics