HW #4

Physics 406

(due 02/27/12)

Reading:

1) Omar (O) Ch. 4
2) Handout 4 : D. Jin, "a Fermi gas of atoms"

Problems:

1. O Pr. 2

2. a) Show that $\sigma = \frac{ne^2\tau}{m}$ in Drude's model, where τ is the collision time.

   b) Using Eq. (4.32) in O, derive the ratio between thermal conductivity K and electrical conductivity σ.

3. Show that the Fermi-Dirac distribution reduces to the classical Boltzmann distribution:

$$f(\mathcal{E}) \simeq e^{-(\mathcal{E}-\mu)/k_B T}$$

$$\mathcal{E} \equiv E, \quad \mu \equiv E_F$$

What are the conditions under which this expansion is valid?

4. Derive the wave function of a free electron confined to a 3D cubic box of side L.

5. 0 Pr. 6

6. 0 Pr. 7

7. Kittel Ch. 6 Pr. 4 (see below)

8. Kittel Ch. 6 Pr. 5 (see below)

9. Please describe the main points in Handout 4. Use a minimum of 4 sentences in your response.

4. ***Fermi gases in astrophysics.*** (a) Given $M_{\odot} = 2 \times 10^{33}$ g for the mass of the Sun, estimate the number of electrons in the Sun. In a white dwarf star this number of electrons may be ionized and contained in a sphere of radius 2×10^{9} cm; find the Fermi energy of the electrons in electron volts. (b) The energy of an electron in the relativistic limit $\epsilon \gg mc^2$ is related to the wavevector as $\epsilon \cong pc = \hbar kc$. Show that the Fermi energy in this limit is $\epsilon_F \approx \hbar c\,(N/V)^{1/3}$, roughly. (c) If the above number of electrons were contained within a pulsar of radius 10 km, show that the Fermi energy would be $\approx 10^8$ eV. This value explains why pulsars are believed to be composed largely of neutrons rather than of protons and electrons, for the energy release in the reaction $n \rightarrow p + e^-$ is only 0.8×10^6 eV, which is not large enough to enable many electrons to form a Fermi sea. The neutron decay proceeds only until the electron concentration builds up enough to create a Fermi level of 0.8×10^6 eV, at which point the neutron, proton, and electron concentrations are in equilibrium.

5. ***Liquid He^3.*** The atom He^3 has spin $\frac{1}{2}$ and is a fermion. The density of liquid He^3 is 0.081 g cm^{-3} near absolute zero. Calculate the Fermi energy ϵ_F and the Fermi temperature T_F.