25/25 HW #1 solutions

Physics 406

(1.) O. Ch. 1, Pr. 1.

The primitive basis vectors are

$$\begin{cases} \vec{a} = \left(\frac{a}{2}, \frac{a}{2}, 0\right), \\ \vec{b} = \left(0, \frac{a}{2}, \frac{a}{2}\right), \\ \vec{c} = \left(\frac{a}{2}, 0, \frac{a}{2}\right) \end{cases}$$

The Bravais lattice is obviously fcc.

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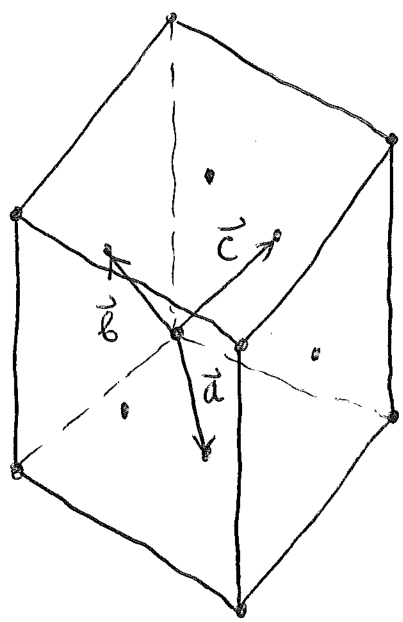

(2.) θ Ch. 1, Pr. 3

Ideal hcp lattice has bottom & top hexagons: top view.

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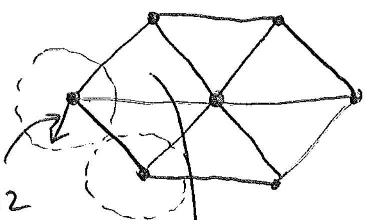
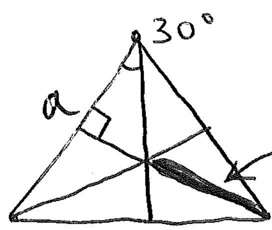


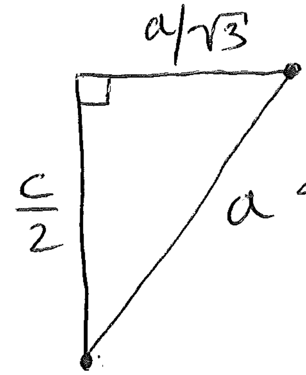
Each vertex is a center of a sphere w/radius $a/2$.

hcp also has an intervening layer of 3 atoms located in the center of the corresponding triangle (if one looks from above):



$\frac{a}{\sqrt{3}}$ (equilateral Δ)

Now, c is constrained by the packing requirement: side view



since 2 spheres of radius $\frac{a}{2}$ must touch

intervening atoms are midway between top & bottom hexagons

So, $$\left(\frac{c}{2}\right)^2 + \frac{a^2}{3} = a^2,$$

$$\left(\frac{c}{a}\right)^2 = \frac{8}{3} \Rightarrow \frac{c}{a} = \sqrt{\frac{8}{3}} \simeq 1.633$$

(3) The bcc lattice with cube side a has volume of a^3 & contains 2 atoms $\left(8 \times \frac{1}{8} + 1 = 2 \frac{\text{atoms}}{\text{unit cell}}\right)$. So, the volume of the primitive cell should be $\frac{a^3}{2}$.

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Similarly, the fcc lattice has 4 atoms $\left(8 \times \frac{1}{8} + 6 \times \frac{1}{2} = 4 \frac{\text{atoms}}{\text{unit cell}}\right)$ & volume a^3. So, the volume of the primitive cell is $\frac{a^3}{4}$.

4. Intercepts are $(3, 2, \bar{1})$.

Reciprocals are $\left(\frac{1}{3}, \frac{1}{2}, \frac{\bar{1}}{2}\right) \Rightarrow$

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$\Rightarrow (2\,3\,\bar{3})$ are the Miller indices.

The direction normal to this plane is $[2\,3\,\bar{3}]$.

5. (a) C_1, C_2 rotations

/, \ reflections

inversion through the center of the figure

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(b) Same as in (a)

(c) Same as in (a) plus

C_4 rotation

|, — reflections