Physics 406, Spring 2011

Midterm II

April 11, 2011

Name solutions

The five problems are worth 20 points each.

Problem	Score
1	
2	
3	
4	
5	
Total	

1. Consider a two-dimensional (2D) film of atoms arranged in a regular simple cubic lattice. The lattice spacing is 2 Å. Let us assume that each atom contributes one "valence" electron that acts as a free electron in the 2D solid.

   a) What is the Fermi energy (in eV) of this 2D solid at T=0? Recall that

$$\frac{\hbar^2}{2m_e} = 3.845\ \text{eV}\cdot\text{Å}^2$$

$$N = \int_0^{E_F} dE\, D(E)\ , \quad \text{where}$$

$$D(E) = 2\frac{dN}{dE} = 2\frac{dN}{dk}\frac{dk}{dE}.$$

In 2D, $N(k) = \dfrac{\pi k^2}{\left(\frac{2\pi}{L}\right)^2} = \dfrac{L^2 k^2}{4\pi}.$

$$E = \frac{\hbar^2 k^2}{2m_e} \Rightarrow \frac{dE}{dk} = \frac{\hbar^2 k}{m_e}.$$

So, $D(E) = 2\dfrac{L^2 k}{2\pi}\dfrac{m_e}{\hbar^2 k} = \dfrac{L^2 m_e}{\pi\hbar^2} = \underline{\underline{\text{const.}}}$

Then $N = \dfrac{L^2 m_e}{\pi\hbar^2}E_F \Rightarrow E_F = \underbrace{\dfrac{N}{L^2}}_{n\ -\ \bar{e}\ \text{concentr'n}}\dfrac{\pi\hbar^2}{m_e}.$

$$n = \frac{1\bar{e}}{(2\,\text{Å})^2} = 0.25\ \text{Å}^{-2}$$

$$E_F = n\frac{\pi\hbar^2}{m_e} = 0.25\ \text{Å}^{-2}\times(2\pi)\times 3.845\,\text{eV}\cdot\text{Å}^2 =$$

$$= \underline{\underline{6.04\ \text{eV}}}.$$

b) Please calculate the average energy per electron in the 2D film, at T=0.

per unit area

$$\langle E \rangle = \int_0^{E_F} dE\, E\, D(E) = \frac{E_F^2}{2} \underbrace{\frac{L^2 m}{\hbar^2 \pi}}_{n/E_F} =$$

$$= \frac{E_F}{2} n .$$

ave. E per $\bar{e}$ is simply

$$\frac{\langle E \rangle}{n} = \frac{E_F}{2}$$, as expected since $D(E) = \text{const}$.

c) At room temperature (T=300K, $k_B T$=0.025 eV), what is the probability that a state 0.05 eV above the Fermi level is occupied by an electron?

This probab. is given by F-D distribution:

$$f(E) = \frac{1}{e^{(E-E_F)/k_B T} + 1} .$$

In this case,

$$f(E) = \frac{1}{e^{\frac{0.05 eV}{0.025 eV}} + 1} = \frac{1}{e^2 + 1} \simeq 0.12 .$$

2. The diagram below shows the electronic band structure of some hypothetical one-dimensional (1D) solid. Will this solid be a metal or an insulator at T=0 if it has:

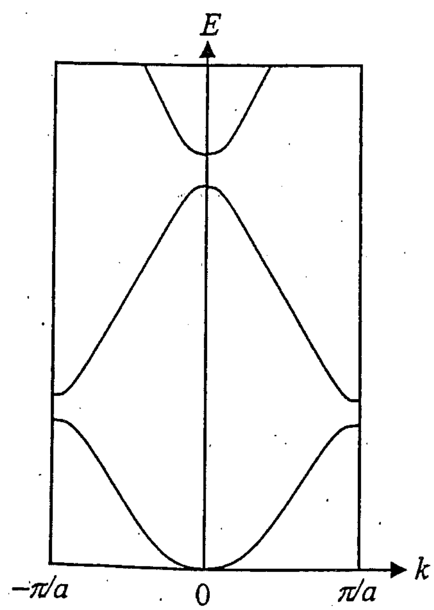
a) 1 electron/primitive unit cell?

b) 2 electrons/primitive unit cell?

c) 3 electrons/primitive unit cell?

d) 4 electrons/primitive unit cell?

Please justify your answers!



metal

$1\, \bar{e}/\text{cell} \Rightarrow N\, \bar{e}'\text{s} \Rightarrow$ 1st band $\frac{1}{2}$ full

$3\, \bar{e}/\text{cell} \Rightarrow 3N\, \bar{e}'\text{s} \Rightarrow$ 2nd band $\frac{1}{2}$ full, 1st band full

$2\, \bar{e}/\text{cell} \Rightarrow 2N\, \bar{e}'\text{s} \Rightarrow$ 1st band full

$4\, \bar{e}/\text{cell} \Rightarrow 4N\, \bar{e}'\text{s} \Rightarrow$ 1st & 2nd bands full

insulator

Key points:

a) Each band accomodates $2N$ $\bar{e}$'s

b) x $\bar{e}$'s / unit cell $\Rightarrow$ Nx total $\bar{e}$'s

Full bands cannot carry electric current. To be a metal, some bands need to be partially occupied.

3. Please describe recent developments in the field of nanoelectronics. Please include some current challenges and future goals in your description, and use a minimum of 4 sentences in your response.

Recent developments include creation of nanometer-scale electronic devices such as transistors & diodes from organic molecules, carbon nanotubes and semiconductor nanowires. The current challenge is to wire some of these components together to create a functional device. Unlike current microelectronics produced in a top-down manufacturing process, nanoelectronics devices will be made in a bottom-up approach. Researchers will make building blocks first which will then either self-assemble or be assembled into larger structures.

4. a) Empirically speaking, what is the difference between a metal and an insulator?

Application of electric field to solid: current in metals, no current in insulators.

b) What is the criterion, based on the energy band theory, for distinguishing a metal from an insulator?

A solid behaves as a metal if some of its bands are partially occupied. In insulators, all bands are either full or empty. It takes $2N$ $\bar{e}$'s to fill a band $\Rightarrow$ insulators must have an even # $\bar{e}$'s per unit cell.

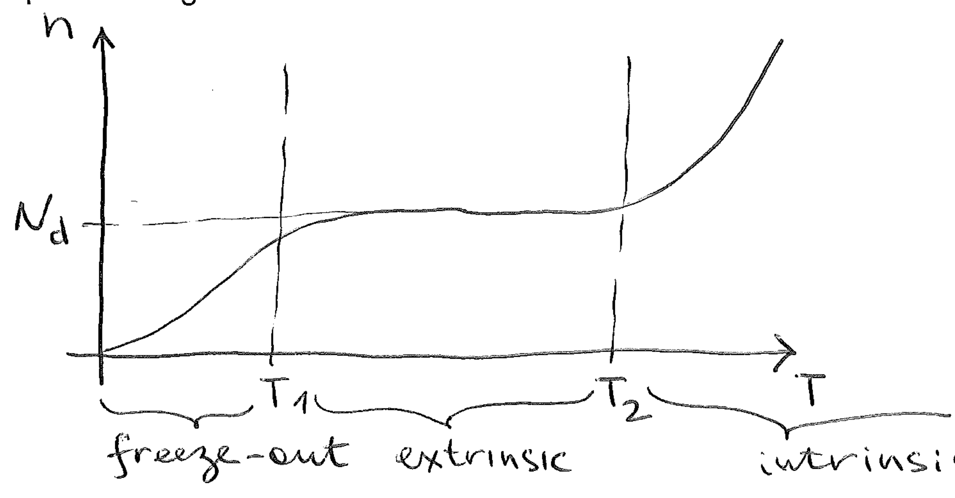
c) From the point of view of band structure, why are some divalent elements such as Be or Mg metals rather than insulators?

In divalent metals some bands overlap, resulting in $\bar{e}$ transfer from one band (filled) to the other band (empty) & thus creating partially filled bands.

5. a) What is a semiconductor and how is it distinctive from a metal and from an insulator?

A semiconductor has a gap of $\lesssim 2$ eV, so that $\bar{e}$'s are thermally excited into the conduction band at high T. Therefore, unlike insulators, semiconductors are conducting at $T \gtrsim 350$ K or so; however the conductivity is substantially lower than with metals due to low carrier concentration.

b) Please sketch n(T) vs. T plot (temperature dependence of the carrier concentration) for an n-type semiconductor. Indicate and explain the three main temperature regimes.



$T < T_1$ donor $\bar{e}$'s "frozen" at impurity sites
$T_1 \approx 100$ K

$T_1 < T < T_2$ extrinsic regime, $n \sim N_d$
all donors ionized, all $\bar{e}$'s from impurities are in conduction band

$T > T_2$ Thermal activation of $\bar{e}$'s: VB → CB becomes dominant

c) Use the Maxwell-Boltzmann approximation to the Fermi-Dirac distribution to compute the electron concentration as a function of Fermi energy and gap energy in the intrinsic semiconductor regime. Use

$$g_e(E) = \frac{1}{2\pi^2}\left(\frac{2m_e}{\hbar^2}\right)^{3/2}(E-E_g)^{1/2}$$

for the density of states on the bottom of the conduction band. E_g - gap energy

Recall that

$$\int_0^{\infty} dx\, x^{1/2} e^{-x} = \frac{\sqrt{\pi}}{2}.$$

Using $f(E) \simeq e^{E_F/k_BT} e^{-E/k_BT}$,

we obtain:

$$n = \int_{E_g}^{E_{c_2}} dE\, f(E)\, g_e(E) =$$

$\emptyset$ energy at top of VB

extend upper limit to $+\infty$ b/c integrand ≈ 0 there

$$= \frac{1}{2\pi^2}\left(\frac{2m_e}{\hbar^2}\right)^{3/2}\int_{E_g}^{+\infty} dE\, e^{E_F/k_BT} e^{-E/k_BT}(E-E_g)^{1/2} =$$

$$= \frac{1}{2\pi^2}\left(\frac{2m_e}{\hbar^2}\right)^{3/2} e^{E_F/k_BT} e^{-E_g/k_BT}(k_BT)^{3/2}\underbrace{\int_0^{+\infty} dx\, x^{1/2} e^{-x}}_{\frac{\sqrt{\pi}}{2}} =$$

$$x = \frac{E-E_g}{k_BT}$$

$$= \frac{1}{4}\left(\frac{2m_e}{\pi\hbar^2}\right)^{3/2}(k_BT)^{3/2} e^{E_F/k_BT} e^{-E_g/k_BT} =$$

$$= 2\left(\frac{m_e k_BT}{2\pi\hbar^2}\right)^{3/2} e^{E_F/k_BT} e^{-E_g/k_BT}.$$