HW #8

Solutions

Physics 406

1. Omar Q. 2

Eq. (10.12):

$$H_c(0) \simeq \left(\frac{2 n k_B^2}{\mu_0 E_F} \right)^{1/2} T_c$$

Recall that in 3D,

$$E_F \sim n^{2/3}$$

Thus $H_c(0) \sim n^{1/6} T_c$, the dependence on n is very weak

$H_c(0) \sim T_c$ for any superconductor

2. Omar Q. 6

The binding energy of a Cooper pair (i.e. the energy required to break it up) is roughly $\Delta(T)$, the gap energy. It is easier to break up the Cooper pair as $T\uparrow$ due to thermal fluctuations. Thus we expect $\Delta(T)$ to decrease with $T\uparrow$ until $T = T_c$. At $T \gtrsim T_c$ Cooper pair formation is no longer energetically favorable.

3. O chapter 10 p. 6.

Perfect diamagnetism is the crucial property of a superconductor (and not perfect conductivity)

$B = 0$ for $T < T_c$ does not follow from perfect conductivity

Ohm's Law $E = \rho j$

$\rho \to 0$ but j finite

$\Downarrow$

$E = 0$

Maxwell's equation

$$\nabla \times E = \frac{\partial B}{\partial t} \quad \Rightarrow \quad \frac{\partial B}{\partial t} = 0$$

(NOT $B = 0$)

A perfect conductor implies that the flux through such a metal cannot change on cooling through the transition, contrary to observation

4. <u>Omar Q.5</u>

London equation:

$$\vec{B} = -\frac{m}{n_s e^2} \vec{\nabla} \times \vec{J}_s \quad , \text{ where}$$

$$\vec{J}_s = -n_s e \vec{V}_s .$$

$$B_y = -\frac{m}{n_s e^2}\left(\overbrace{\frac{\partial J_{s,x}}{\partial z}}^{=0} - \frac{\partial J_{s,z}}{\partial x}\right) =$$

$$= \frac{m}{n_s e^2} \frac{\partial J_{s,z}}{\partial x} .$$

But $B_y(x) = B_y(0)\, e^{-x/\lambda}$, so that

$$J_{s,z}(x) = \left(\frac{n_s e^2}{m}\right)(-\lambda)\, B_y(x) =$$

$$= -\left(\frac{n_s^2 e^4}{m^2} \frac{m}{\mu_0 n_s e^2}\right)^{1/2} B_y(x) =$$

$$= -\left(\frac{n_s e^2}{\mu_0 m}\right)^{1/2} B_y(x) \equiv -J_{s,z}(0)\, e^{-x/\lambda} .$$

5. Key points on the discovery and properties of MgB_2

- Original idea by Akimitsu et al : investigation of ternary compound with two light atoms (Mg and B) and a not-so-heavy one (Ti) with a large density of states and therefore a large T_c.

- Discovered that superconductivity near $T \sim 40$ K due to binary compound MgB_2.

- MgB_2 wires can be made relatively easily from boron filaments

- Isotope effect in MgB_2 superconductivity points to underlying electron-phonon mechanism

- Combination of very high critical field and low normal-state resistivity make MgB_2 an attractive material for a superconducting magnet

- Highly anisotropic critical field

- Two superconducting gaps ?