HW #6
solutions

Physics 406

1. θ Ch. 6, Q. 1

ē's participating in the tetrahedral bond are in the valence band, from the point of view of the band structure. The "band" view and the "bond" view do not contradict each other because:

a) Bloch function can be both localized. ~~and~~ and periodic. ~~manner~~

b) ē's are not "assigned" to a particular bond in a crystal, but can contribute to any bond, in agreement with the Bloch function periodicity

2. θ Ch. 6, Q. 4

Breaking of a bond corresponds to an ē leaving the valence band and entering the conductance band; there is now a hole in the valence band.

3. O chapter 6 P 2

a) $n = p$ for Si at $T = 300\,K$

$$m_e = 0.7\ m_0$$

$$m_h = m_0 \qquad E_g = 1.1\ eV$$

$$n = p = 2\left(\frac{k_B T}{2\pi\hbar^2}\right)^{3/2}(m_e m_h)^{3/4}\, e^{-E_g/2 k_B T}$$

$$\left(\frac{k_B T}{2\pi\hbar^2}\right)^{3/2} = \left(\frac{1.4\times 10^{-23}\cdot 300}{2\pi\,(1.05\times 10^{-34})^2}\right)^{3/2}$$

$$= \left[\frac{(1.4)(3)}{2\pi\,(1.05)^2}\ \underbrace{10^{-21}\ 10^{68}}_{10^{47}}\right]^{3/2}$$

$$= \left[\frac{(1.4)\,(3)}{(2\pi)(10)\,(1.05)^2}\right]^{3/2} 10^{72}$$

$$(m_e m_h)^{3/4} = \left((0.7)\,(9.1\times 10^{-31})^2\right)^{3/4}$$

$$= \left[\frac{(0.7)\,(9.1)^2}{100}\right]^{3/4} 10^{-45}$$

$$\frac{E_g}{2k_BT} = \frac{(1.1)(1.6\times 10^{-19})}{2(1.4\times 10^{-23})(300)}$$

$$= \frac{(1.1)(1.6)}{(2)(1.4)(3)} \underbrace{10^{-19}\ 10^{21}}_{10^{2}}$$

$$= .2095\times 10^{2} = 20.95 \sim 21$$

$$e^{-E_g/2k_BT} = 7.6\times 10^{-10}$$

$$\Downarrow$$

$$n = \underbrace{(2)(7.6)}_{(15.2)} \underbrace{\left[\frac{(1.4)(3)}{(2\pi)(10)(1.05)^2}\right]^{3/2}}_{(0.0149)} \underbrace{\left[\frac{(0.7)(9.1)^2}{100}\right]^{3}}_{(0.66)}$$

$$\times \underbrace{10^{72}\cdot 10^{-45}\cdot 10^{-10}}_{10^{17}}$$

$$= .15 \times 10^{17}\ /m^3$$

$$= 1.5\times 10^{16} \text{ electrons}/m^3$$

h) E_F

$$E_F = \frac{1}{2} E_g + \frac{3}{4} k_B T \log \frac{m_h}{m_e}$$

$$= \frac{1}{2} (1.1 \text{ eV}) + \frac{3}{4} (.026 \text{ eV}) \log \left(\frac{1}{.7}\right)$$

$$k_B T = \frac{(1.4 \times 10^{-23} \cdot 300)}{1.6 \times 10^{-19}} \text{ eV} = \frac{(1.4)(3)}{1.6} \cdot 10^{-2}$$

$$= 2.6 \times 10^{-2} \text{ eV}$$

$$= .026 \text{ eV}$$

$$\Downarrow$$

$$\boxed{E_F = .553 \text{ eV}}$$

4 O chapter 6 P 5

Si $N_d = 1 \times 10^{23}\ m^{-3}$ $(T = 300\ K)$

a) In # 5 (O chapter 6 p. 2) we just determined

$$n \sim 10^{16}\ m^{-3}$$

$$\Downarrow$$

$$N_d \gg n.$$

b) $$n = N_d = 2\left(\frac{m_e k_B T}{2\pi \hbar^2}\right)^{3/2} e^{E_F/k_BT}\, e^{-E_g/}$$

$$\Downarrow$$

$$E_F = E_g + k_B T \ln\left[\left(\frac{N_d}{2}\right)\left(\frac{2\pi\hbar^2}{m_e k_B T}\right)^{3/2}\right]$$

From our previous work on this HW.

$E_g = 1.1$

$(k_B T)_{T=300K} = 0.026\ eV$

$$\left(\frac{2\pi\hbar^2}{m_e k_B T}\right)^{3/2} = \left(\frac{(2\pi)\cancel{(10)}(1.05)^2}{(.7)(9.1)\times(1.4)(3)\cancel{(10)}}\right)^{3/2}$$

$m_e \to 0.7\, m_0$

$$\times \underbrace{10^{48} \times 10^{-72}}_{10^{-24}}$$

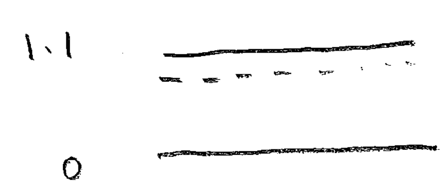
$$\frac{N_d}{2} = \frac{10^{23}}{2}$$

$$\ln\left[\quad\right] = \ln\left[(10^{-1})\,\frac{1}{2}\left(\frac{(2\pi)(1.05)^2}{(.7)(9.1)(1.4)(3)}\right)^{3/2}\right]$$

$$\Downarrow$$

$$E_F = (1.1) + 0.026\,\underbrace{\ln\left[\frac{1}{20}\left(\frac{(2\pi)(1.05)^2}{(.7)(9.1)(1.4)(3)}\right)^{3/2}\right]}_{(-5.02)}$$

$$\boxed{E_F = +0.969 \text{ eV}} \checkmark$$



We expect

$E_F < E_g$ ✓

c) $N_a = 6 \times 10^{21} \ m^{-3}$

Since $N_d \gg N_a$ the Fermi level will only be slightly shifted by the introduction of these acceptor impurities

In particular now we'll have

$$n \sim \tilde{N}_d \sim N_d - N_a$$

$$\Downarrow$$

$$E_F \sim E_g + k_B T \ln \underbrace{\left[\left(\frac{\tilde{N}_d}{2} \right) \left(\frac{2\pi\hbar^2}{m_e k_B T} \right) \right]^{3/2}}_{<0}$$

$$\tilde{N}_d < N_d$$

$$\Downarrow$$

E_F will be slightly higher than before ($N_a = 0$, $N_d \sim 10^{23}$).

5. 0 chapter 6 p. 6.

Si $\mu_e = 1350\ cm^2/V\text{-}s$ $\qquad m_h = m_0$

$\mu_h = 475\ cm^2/V\text{-}s$ $\qquad m_e = (0.7)\, m_0$

a) Electron/hole lifetimes

$$\mu_e = \frac{e\tau_e}{m_e} \rightarrow \tau_e = \frac{\mu_e\, \overset{.7\,m_0}{m_e}}{e}$$

$$= \frac{(1350 \times 10^{-4})(.7)(9 \times 10^{-31})}{(1.6 \times 10^{-19})}$$

$$= \frac{(1.350)\, 9\, (.7)}{1.6}\ 10^{-13}\ s$$

$$\boxed{\tau_e = 5.3 \times 10^{-13}\ s}$$

$$\tau_h = \frac{\mu_h m_h}{e} = \frac{\mu_h}{\mu_e}\frac{m_h}{m_e}\tau_e$$

$$= \frac{475}{1350} \cdot \frac{1}{.7}\,(5.3 \times 10^{-13}\, s)$$

$$\boxed{\tau_h = 2.7 \times 10^{-13}\ s}$$

b) $\sigma = ne(\mu_e + \mu_h)$

$$= (1.5 \times 10^{16}) \times (1.6 \times 10^{-19}) \left\{ (1350 + 475) \times 10^{-4} \right.$$

$\downarrow$ from earlier problem

$$= (1.5)(1.6)(1825) \cdot 10^{-7}$$

$$\sigma = 4.38 \times 10^{-4} \ (\text{ohm-m})^{-1}$$

c) T-dependance of σ

The main T-dependance of σ will come from n

$$\sigma \simeq f(T)\, e^{-E_g/2k_B T}$$

$$\Downarrow$$

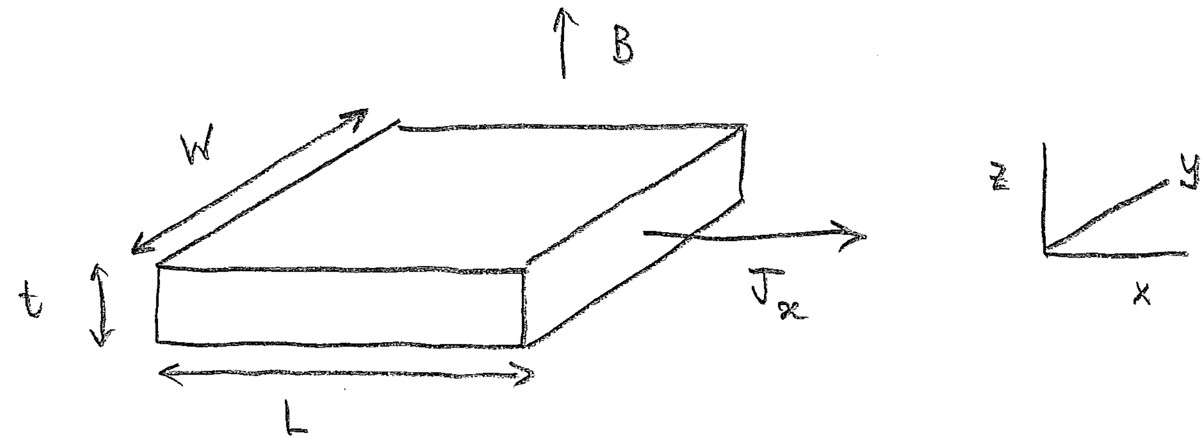
$$\log \sigma = \log \underbrace{f(T)}_{\substack{\Downarrow \\ \text{assumed} \\ \text{weakly T-dep.}}} - \frac{E_g}{2k_B T}$$

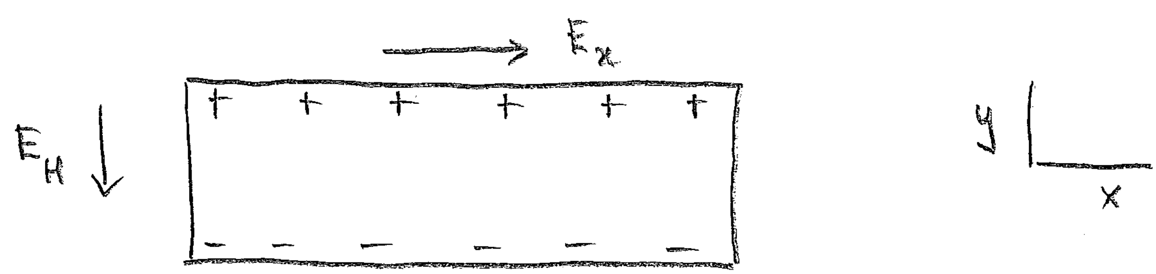
For Si $\quad \dfrac{E_g}{2k_B} \doteq \dfrac{(1.1 \text{ eV})}{(2)(8.6 \times 10^{-5} \text{ eV/K})}$

$$= 6.3 \times 10^3 \quad 1/T$$

Phonon-scattering will play a role in $\mu(T)$, but this is weak compared to the T-dependence of n.

6. O chapter 6 p 8





$L = 5$ cm $\qquad$ $B = 0.6$ Wb/m^2

$t = 1$ mm $\qquad$ $V_H = 8$ mV

$W = 0.5$ cm $\qquad$ $J_x = 10$ mA

$$R_H \equiv \frac{E_y}{j_x B_z}$$

We are asked to solve for

a) Mobility of carrier $\Rightarrow$ However E_x not given!

b) carrier concentration

$\Downarrow$

solve for R_H, p.

We will assume a dominant carrier type $\Rightarrow$

sign of R_H will tell us if hole or electron

$$E_y = \frac{V_H}{W} = \frac{8\ \text{mV}}{.5\ \text{cm}} = \frac{8\ \text{mV}}{.005\ \text{m}}$$

$$= \frac{8}{5}\ \frac{10^{-3}}{10^{-3}}\ \frac{V}{m}$$

$$= 1.6\ V/m$$

$$R_H = \frac{E_y}{J_x B_z} = \frac{1.6}{10^{-2}\ (0.6)}$$

$$= \frac{1.6}{6} \times 10^{3}$$

$$\boxed{R_H = 2.6 \times 10^{2}\ \frac{V-m^{3}}{\text{amp-weber}}}$$

$R_H > 0 \Rightarrow$ p-type

(holes dominate)

$$R_H = \frac{1}{pe} \Rightarrow p = \frac{1}{R_H e}$$

$$p = \frac{1}{(2.6 \times 10^2)(1.6 \times 10^{-19})}$$

$$= \frac{1}{(2.6)(1.6)} \quad 10^{17}$$

$$\boxed{p = 2.4 \times 10^{16} \text{ holes} / m^3}$$

Mobility

$$\mu_H = \sigma R_H$$

$$\sigma = \frac{J_x}{E_x}$$

E_x not given $\Rightarrow$ not enough information to determine σ!

(Sorry!)

7. O chapter 6 p. 11

a) Density of states for ellipsoidal energy surface

We have been writing

$$\frac{dN}{dE} = \frac{dN}{dk} \frac{dk}{dE}$$

However we can also write

$$\frac{dN}{dE} = \frac{dN}{dV} \frac{dV}{dE}$$

· spherical energy surface

$$V = \frac{4\pi}{3} k^3 = \frac{4\pi}{3} \left(\frac{2mE}{\hbar^2} \right)^{3/2}$$

$$E = \frac{\hbar^2}{2m} k^2 \Rightarrow k = \sqrt{\frac{2mE}{\hbar^2}}$$

$$\frac{dN}{dE} = \underbrace{\frac{1}{(2\pi)^3}}_{\frac{dN}{dV}} \cdot \underbrace{2\pi \left(\frac{2m}{\hbar^2} \right)^{3/2} E^{1/2}}_{\frac{dV}{dE}}$$

$$\frac{dN}{dE} = \frac{1}{(2\pi)^2} \left(\frac{2m}{\hbar^2} \right)^{3/2} E^{1/2}$$

· ellipsoidal energy surface

$$V = \frac{4\pi}{3} \, k_\perp^2 \, k_z$$

$$k_\perp = \sqrt{\frac{2E}{\hbar^2} \, m_\perp}$$

$$k_z = \sqrt{\frac{2E}{\hbar^2} \, m_z}$$

$$\Downarrow$$

$$\boxed{\frac{dN}{dE} = \frac{1}{(2\pi)^2} \cdot \left(\frac{2}{\hbar^2}\right)^{3/2} \left(m_\perp^2 \, m_z\right)^{1/2} \, E^{1/2}}$$

b) m_d for Ge

According to Table 6.1 for electrons in Ge

$$m_\ell = 1.6 \; m_0$$

$$m_t = 0.08 \; m_0$$

$$m_d = \left(m_t^2 \, m_\ell\right)^{1/3} = \left((.08)^2 (1.6)\right)^{1/3} m_0$$

$$= .22 \; m_0$$

18. Developments in the area of nanowires / nanoelectronics

- Molecular electronics where switching mechanism involves oxidation reduction
- Use of carbon nanotubes for electronics exploiting their good current-carrying capacity and possible overlap with physics of silicon electronics
- Single-electron transistors composed of metallic carbon nanotubes
- Computation with DNA molecules exploiting the complementary nature of their strands
- Self-assembly of nanowires and nanoarrays
- use as ultrasensitive detectors to detect gas molecules + range of biological compounds