1. 0 chapter 4, Q 8

\[ R_H \sim \frac{1}{N} \]

The Hall constant \( R_H \) is defined as

\[ R_H = \frac{E_H}{J_x B} \]

where \( E_H \) is the Hall field, \( B \) is the applied field, and \( J_x \) is an electric current. Current is charge/\( \text{unit time} \) and thus is proportional to the concentration of electrons \( N \Rightarrow R_H \sim \frac{1}{N} \).

2. 0 chapter 4, Q 9

Positive charge carriers will mean \( J = N (+e) v_x \)

\[ \Rightarrow \]

\[ R_H = \frac{E_H}{\frac{1}{+e}} > 0. \]
Here we must distinguish between core (localized) and valence (delocalized) electrons. The combination of the core electrons and the ions leads to a weak pseudopotential; the core electrons thus "screen" the ions. Bloch's theorem tells us that the periodic nature of this resulting potential results in valence electrons with propagating wavefunctions. Thus there is no paradox - rather there are two types of electrons.
4. 0 chap. 4 P 10

\[ v_c = \frac{w_c}{2\pi} = 2.8 \text{ GHz} \quad \text{from } m = m^* \]

\[ v_c = 24 \text{ GHz} \quad \Rightarrow \quad B = \frac{24}{2.8} \text{ kGauss} \]

\[ B = 8.6 \text{ kGauss} \]
5. Kittel chapter 6 P 6

\[ m \left( \frac{dv}{dt} + \frac{v}{\tau} \right) = -eE \]

Let \( v = v_0 \, e^{-i\omega t} \)

\[ E = E_0 \, e^{-i\omega t} \]

\[ \downarrow \]

\[ (-i\omega + \frac{1}{\tau}) \, v_0 = -eE_0 / m \]

\[ \downarrow \]

\[ v_0 = \frac{-eE_0 / m}{-i\omega + \frac{1}{\tau}} = \frac{-eE}{m} \frac{1+i\omega\tau}{1+(\tau\omega)^2} \]

\[ j = n(-e) \, v = \frac{ne^2 \tau}{m} \frac{1+i\omega\tau}{1+(\tau\omega)^2} E \]

\[ \equiv \sigma E \]

\[ \downarrow \]

\[ o(\omega) = o(0) \frac{1+i\omega\tau}{1+(\tau\omega)^2} \]

a) \[
\begin{array}{c}
\sqrt{\frac{3}{2}} \frac{\alpha}{a} \\
\frac{\pi}{a}
\end{array}
\]

\[k_{\text{energ}} = \sqrt{\frac{3}{2}} \ k \ \text{center}
\]

\[E \sim k^2 \Rightarrow E_{\text{energ}} = 2
\]

\[\frac{\sqrt{3}}{E_{\text{center}}}
\]

b) \[
\begin{array}{c}
k_{\text{energ}} = \sqrt{3} \ k \ \text{center}
\end{array}
\]

\[E \sim k^2 \Rightarrow E_{\text{energ}} = 3
\]

\[\frac{\sqrt{3}}{E_{\text{center}}}
\]

c) If \(E_{\text{gap}} < E_{\text{energ}} - E_{\text{center}}\)

Electrons will find it energetically more favorable to occupy center states in the 2nd zone rather
than filling up corner states in the 1st Brillouin zone. Under these conditions, divalent elements could be metals and not insulators.

The central equation is

\[(\lambda_k - \epsilon) c_k + \sum_{G} U_G c_{k-G} = 0\]

We need to find \(U_G\):

\[U(x) = \sum_{G} U_G \, e^{iG \cdot \vec{x}}\]

\[\downarrow\]

\[U_G = \frac{1}{a^2} \int_{\text{all}} U(x) \, e^{-i\vec{G} \cdot \vec{x}} \, dx \, dy\]

\(\vec{G} = \left( \frac{2\pi}{a}, \frac{2\pi}{a} \right)\)

Let \(x_i = \begin{cases} x & i = 1 \\ y & i = 2 \end{cases}\)
Then

\[
U_g = -\frac{4U}{a^2} \prod_{i=1}^{2} \frac{2}{i} \int_0^a dx_i \cos 2\pi x_i \frac{e^{-i2\pi x_i}}{a} - \frac{-i2\pi x_i}{a}
\]

\[
= -\frac{4U}{a^2} \prod_{i=1}^{2} \frac{2}{i} \int_0^a dx_i \left[ \frac{e^{-i2\pi x_i}}{a} + e^{-i2\pi x_i} \right]
\]

\[
= -\frac{4U}{a^2} \prod_{i=1}^{2} \frac{2}{i} \int_0^a dx_i \left[ \frac{e^{-i4\pi x_i}}{a} \right]
\]

\[
= -\frac{U}{a^2} \left[ a \right] \left[ a \right] \left[ a \right] \left[ a \right]
\]

\[
= -U \quad \Rightarrow \quad U_g = -U
\]
Then the central equations can be written in matrix form

\[
\begin{bmatrix}
\lambda_k - \epsilon & -u \\
-u & \lambda_k - \epsilon
\end{bmatrix}
\begin{bmatrix}
e_k \\
e_{k-q}
\end{bmatrix} = 0.
\]

\[\det \begin{bmatrix}
\lambda_k - \epsilon & -u \\
-u & \lambda_k - \epsilon
\end{bmatrix} = 0 \implies (\lambda_k - \epsilon)(\lambda_k - \epsilon) - u^2 = 0.
\]

\[\lambda_k = \frac{k^2}{2m}.
\]

\[\hat{\mathbf{F}} = \left( \frac{\pi}{a}, \frac{\pi}{a} \right) \implies \hat{\mathbf{k}} - \hat{\mathbf{G}} = \left( \frac{-\pi}{a}, \frac{-\pi}{a} \right)
\]

\[\hat{\mathbf{G}} = \left( \frac{2\pi}{a}, \frac{2\pi}{a} \right)
\]

\[\lambda_k = \lambda_{k-q}
\]
Therefore the equation we need to solve is

\[(\lambda - \varepsilon)^2 = u^2\]

\[\downarrow\]

\[\varepsilon_{\pm} = \lambda \pm u\]

\[\downarrow\]

\[\gamma_{\text{ap}} = \varepsilon_{+} - \varepsilon_{-} = 2u.\]
8. O chapter 5 P 12.

a) 1d  \[ k = \left( \frac{2\pi}{L} \right)^n \]

\[ N = \frac{kL}{2\pi} \Rightarrow \frac{dN}{dk} = \frac{L}{2\pi} \]

\[ g(E) = \frac{dN}{dE} = \frac{dN}{dk} \frac{dk}{dE} \]

\[ g(E) = \frac{L}{2\pi} \frac{dE}{dk} \]

b) For 1d the tight-binding model yields

\[ E(k) = E_0 + 4\gamma \sin^2 \frac{ka}{2} \]
\[
\frac{dE}{dk} = 4\pi 2 \sin \frac{ka}{2} \left( \frac{a}{2} \right) \cos \frac{ka}{2}.
\]

\[
\frac{dE}{dk} = 4\pi a \sin \frac{ka}{2} \cos \frac{ka}{2}
\]

\[
g(E) = \frac{1/2\pi}{4\pi a \sin \frac{ka}{2} \cos \frac{ka}{2}}
\]

\[
\text{Limits}
\]

\[
k \rightarrow 0 \quad g(E) \sim \frac{1}{k} \sim E^{-1/2}
\]

\[
k \rightarrow \frac{\pi}{a} \quad g(E) \rightarrow \infty
\]
a) Free Electron Model

\[ n = \frac{2}{(2\pi)^3} \frac{4}{3} \pi k_F^3 \]

\[ \downarrow \]

\[ n = \frac{1}{3\pi^2} k_F^3 \Rightarrow k_F = (3\pi^2 n)^{1/3} \]

b) Fermi sphere touches face of 1st BZ of fcc lattice

where

\[ k_F = k_i \]

Here \( k_i \) is the shortest distance from the origin to the 1BZ boundary \( \Rightarrow \)

\[ k_i = \frac{1}{2} a_i^* \text{ in fcc} \]

Reciprocal lattice vectors in fcc lattice

\[ a^* = \frac{2\pi}{a} (1, -1, 1) \]

\[ b^* = \frac{2\pi}{a} (1, 1, -1) \]

\[ c^* = \frac{2\pi}{a} (1, 1, 1) \]
Then
\[ k_i = \frac{1}{2} \frac{2\pi}{a} (1^2 + 1^2 + 1^2)^{\frac{1}{2}} = \frac{\sqrt{3} \pi}{a}. \]

For fcc lattice
\[ n = \frac{4 \text{ atoms}}{a^3} \times \frac{n}{na} \]

\[ k_F = k_i \]

\[ \left( 3\pi^2 \frac{4}{a^3} \frac{n}{na} \right)^{\frac{1}{3}} = \sqrt{3} \frac{\pi}{a} \]

\[ \left( \frac{12}{\pi} \right)^{\frac{1}{3}} \left( \frac{n}{na} \right)^{\frac{1}{3}} \]

\[ \downarrow \]

\[ \frac{n}{na} = \frac{\sqrt{3} \pi}{12} \frac{4}{4} \]

\[ \frac{n}{na} = \frac{\sqrt{3} \pi}{4} = 1.36. \]
Zn divalent
Cu monovalent

\[ n = \frac{4}{a^3} \{ (1 - \alpha) (1) + \alpha (2) \} \]

\[ n = \frac{4}{a^3} \{ 1 + \alpha \} \]

\[ k_F = k_v \text{ for FS to touch face of 1st BZ} \]

\[ (3\pi^2 n)^{1/3} = \sqrt{3} \frac{\pi}{a} \]

\[ \left( \frac{12}{\pi} \right)^{1/3} (1 + \alpha)^{1/3} = \sqrt{3} \]

\[ (1 + \alpha) = \frac{\pi}{12} 3 \sqrt{3} \]

\[ \alpha = \frac{\pi \sqrt{3}}{4} - 1 = 0.36 \]
Concentration of Zn = 36.70
Concentration of Cu = 64.90.

Atomic ratios

\[ X = \frac{x_{Zn}}{A_{Zn}} = \frac{36}{65.4 \text{ g/mol}} = 0.55 \]

\[ Y = \frac{64}{A_{Cu}} = \frac{64}{63.5 \text{ g/mol}} = 1.007 \]

\[ \frac{Cu}{1.007} \frac{Zn}{0.55} \]

\[ \downarrow \text{ Divide each by 0.55} \]

\[ \frac{Zn}{1} \frac{Cu}{1.832} \]
Zn concentration = 36 \, \text{\%} = c_{Zn}

Cu concentration = 64 \, \text{\%} = c_{Cu}

Atomic radii:

\[
X = \frac{c_{Zn}}{A_{Zn}} = \frac{36}{65.4} = 0.55
\]

\[
y = \frac{c_{Cu}}{A_{Cu}} = \frac{64}{63.5} = 1.007
\]

We can therefore write this alloy as

Zn$_{0.55}$ Cu$_{1.007}$

\[\downarrow \text{\% as (dividing by 0.55)}\]

Zn$_{4}$ Cu$_{1.832}$

\[\downarrow \]

Zn$_{5}$ Cu$_{9}$

(since formula must have integer # of constituent atoms)
Developments in the field of photonic crystals (based on Handout 6)

Idea: Develop periodic structures of materials with suitable refractive indices such that incoming light of a particular wavelength results in standing waves due to reflection and refraction.

Hope: Analog of doping in semiconductors can be achieved by slightly modifying structure (e.g., extra holes).

Challenges:
- Each structural unit must produce reflected and refracted waves that cancel out incoming waves travelling in all directions.
- Identification of appropriate structures and materials with necessary refractive indices.

Impact:
- Nanolasers
- High capacity optical fibres